

# Computer Mathematics

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## 1. Taylor.

### 1.1. Value

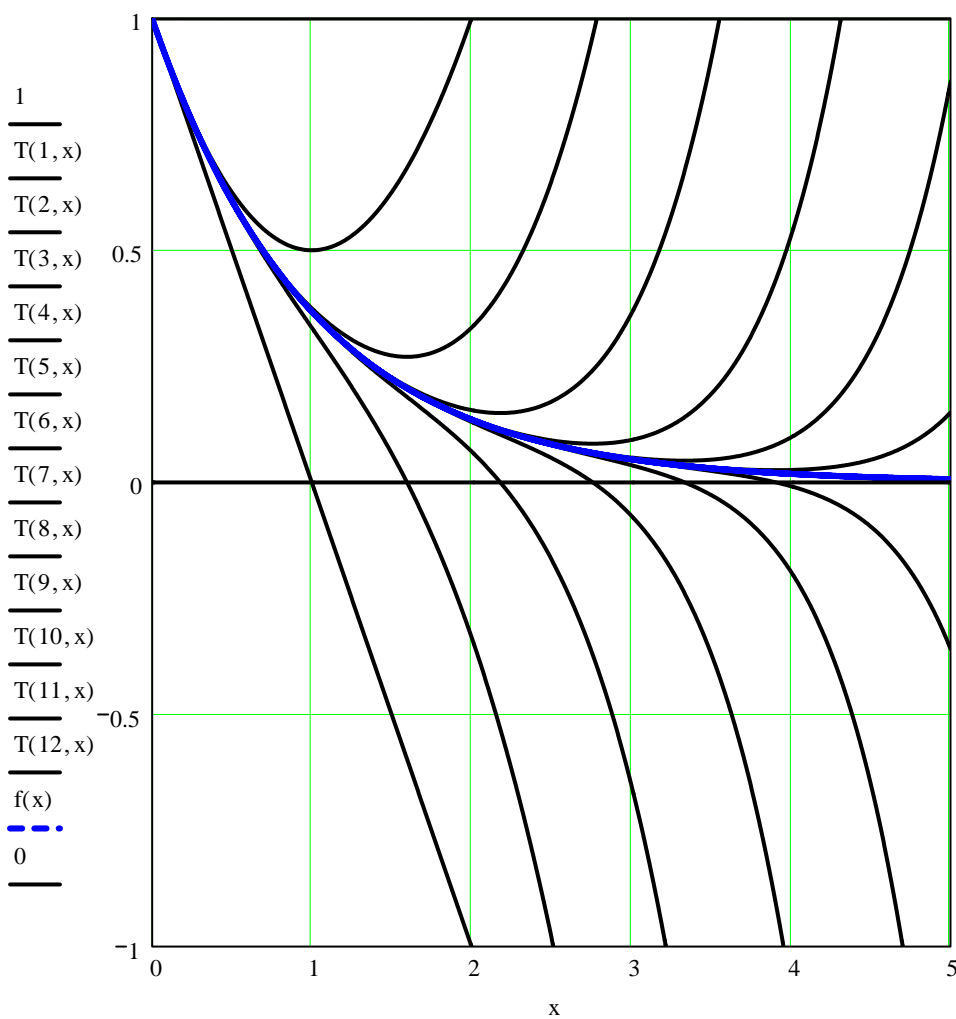
$$f(x) := e^{-x}$$

$$f15(n, x) := \sum_{k=0}^n \left[ (-1)^k \cdot \frac{x^k}{k!} \right]$$

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T(n, x) :=
  Tempo ← 1
  for m ∈ n, n - 1 .. 1
    t ← 1 -  $\frac{x \cdot \text{Tempo}}{m}$ 
    Tempo ← t
  return Tempo

```



$n := 0..5$

$Xa_n := 1 + 1 \cdot n$

$Xb_n := Xa_{\text{last}(Xa)} + 1 \cdot n$

$ma := 1..30$

$mb := 1..33$

$Ra_{ma, n+1} := T(ma, Xa_n) - f(Xa_n)$

$Ra_{0, n+1} := Xa_n$

$Ra_{ma, 0} := ma$

$Rb_{mb, n+1} := T(mb, Xb_n) - f(Xb_n)$

$Rb_{0, n+1} := Xb_n$

$Rb_{mb, 0} := mb$

|      | 0  | 1                        | 2                        | 3                        | 4                        | 5                        | 6                        |
|------|----|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|      | 0  |                          |                          |                          |                          |                          |                          |
|      | 1  | -0.368                   | -1.135                   | -2.05                    | -3.018                   | -4.007                   | -5.002                   |
|      | 2  | 0.132                    | 0.865                    | 2.45                     | 4.982                    | 8.493                    | 12.998                   |
|      | 3  | -0.035                   | -0.469                   | -2.05                    | -5.685                   | -12.34                   | -23.002                  |
|      | 4  | 0.007                    | 0.198                    | 1.325                    | 4.982                    | 13.702                   | 30.998                   |
|      | 5  | -0.001                   | -0.069                   | -0.7                     | -3.552                   | -12.34                   | -33.802                  |
|      | 6  | 0                        | 0.02                     | 0.313                    | 2.137                    | 9.361                    | 30.998                   |
|      | 7  | $-2.23 \times 10^{-5}$   | -0.005                   | -0.121                   | -1.114                   | -6.14                    | -24.545                  |
|      | 8  | $2.503 \times 10^{-6}$   | 0.001                    | 0.042                    | 0.512                    | 3.548                    | 17.112                   |
|      | 9  | $-2.525 \times 10^{-7}$  | 0                        | -0.013                   | -0.211                   | -1.834                   | -10.66                   |
|      | 10 | $2.311 \times 10^{-8}$   | $4.391 \times 10^{-5}$   | 0.004                    | 0.078                    | 0.857                    | 6.003                    |
|      | 11 | $-1.938 \times 10^{-9}$  | $-7.401 \times 10^{-6}$  | -0.001                   | -0.027                   | -0.366                   | -3.086                   |
|      | 12 | $1.498 \times 10^{-10}$  | $1.15 \times 10^{-6}$    | 0                        | 0.008                    | 0.144                    | 1.459                    |
|      | 13 | $-1.075 \times 10^{-11}$ | $-1.657 \times 10^{-7}$  | $-4.564 \times 10^{-5}$  | -0.002                   | -0.052                   | -0.639                   |
|      | 14 | $7.195 \times 10^{-13}$  | $2.226 \times 10^{-8}$   | $9.226 \times 10^{-6}$   | 0.001                    | 0.018                    | 0.26                     |
|      | 15 | $-4.519 \times 10^{-14}$ | $-2.801 \times 10^{-9}$  | $-1.747 \times 10^{-6}$  | 0                        | -0.006                   | -0.099                   |
| Ra = | 16 | $2.665 \times 10^{-15}$  | $3.315 \times 10^{-10}$  | $3.109 \times 10^{-7}$   | $3.945 \times 10^{-5}$   | 0.002                    | 0.036                    |
|      | 17 | 0                        | $-3.703 \times 10^{-11}$ | $-5.221 \times 10^{-8}$  | $-8.853 \times 10^{-6}$  | 0                        | -0.012                   |
|      | 18 | 0                        | $3.917 \times 10^{-12}$  | $8.301 \times 10^{-9}$   | $1.88 \times 10^{-6}$    | 0                        | 0.004                    |
|      | 19 | 0                        | $-3.934 \times 10^{-13}$ | $-1.253 \times 10^{-9}$  | $-3.792 \times 10^{-7}$  | $-3.161 \times 10^{-5}$  | -0.001                   |
|      | 20 | 0                        | $3.741 \times 10^{-14}$  | $1.801 \times 10^{-10}$  | $7.276 \times 10^{-8}$   | $7.593 \times 10^{-6}$   | 0                        |
|      | 21 | 0                        | $-3.442 \times 10^{-15}$ | $-2.468 \times 10^{-11}$ | $-1.332 \times 10^{-8}$  | $-1.74 \times 10^{-6}$   | $-9.27 \times 10^{-5}$   |
|      | 22 | 0                        | 0                        | $3.236 \times 10^{-12}$  | $2.331 \times 10^{-9}$   | $3.812 \times 10^{-7}$   | $2.44 \times 10^{-5}$    |
|      | 23 | 0                        | 0                        | $-4.063 \times 10^{-13}$ | $-3.908 \times 10^{-10}$ | $-7.997 \times 10^{-8}$  | $-6.15 \times 10^{-6}$   |
|      | 24 | 0                        | 0                        | $4.896 \times 10^{-14}$  | $6.287 \times 10^{-11}$  | $1.61 \times 10^{-8}$    | $1.487 \times 10^{-6}$   |
|      | 25 | 0                        | 0                        | $-5.662 \times 10^{-15}$ | $-9.72 \times 10^{-12}$  | $-3.115 \times 10^{-9}$  | $-3.457 \times 10^{-7}$  |
|      | 26 | 0                        | 0                        | 0                        | $1.446 \times 10^{-12}$  | $5.801 \times 10^{-10}$  | $7.732 \times 10^{-8}$   |
|      | 27 | 0                        | 0                        | 0                        | $-2.078 \times 10^{-13}$ | $-1.041 \times 10^{-10}$ | $-1.667 \times 10^{-8}$  |
|      | 28 | 0                        | 0                        | 0                        | $2.846 \times 10^{-14}$  | $1.805 \times 10^{-11}$  | $3.47 \times 10^{-9}$    |
|      | 29 | 0                        | 0                        | 0                        | $-3.518 \times 10^{-15}$ | $-3.023 \times 10^{-12}$ | $-6.977 \times 10^{-10}$ |
|      | 30 | 0                        | 0                        | 0                        | 0                        | $4.857 \times 10^{-13}$  | $1.357 \times 10^{-10}$  |

|      | 0  | 6                         | 7                         | 8                        | 9                        | 10                       | 11                       |
|------|----|---------------------------|---------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|      | 1  | -5.0025                   | -6.0009                   | -7.0003                  | -8.0001                  | -9                       | -10                      |
|      | 2  | 12.9975                   | 18.4991                   | 24.9997                  | 32.4999                  | 41                       | 50.5                     |
|      | 3  | -23.0025                  | -38.6676                  | -60.3337                 | -89.0001                 | -125.6667                | -171.3334                |
|      | 4  | 30.9975                   | 61.3741                   | 110.333                  | 184.3749                 | 291                      | 438.7083                 |
|      | 5  | -33.8025                  | -78.6842                  | -162.7337                | -307.7001                | -542.3334                | -903.3834                |
|      | 6  | 30.9975                   | 84.7171                   | 201.3552                 | 430.4124                 | 846.5555                 | 1557.118                 |
|      | 7  | -24.5453                  | -78.6842                  | -214.7464                | -518.5894                | -1137.5715               | -2309.3841               |
|      | 8  | 17.1118                   | 64.292                    | 201.3552                 | 549.0376                 | 1342.5873                | 3007.0564                |
|      | 9  | -10.6596                  | -46.9118                  | -168.5129                | -518.5894                | -1413.1447               | -3490.8154               |
|      | 10 | 6.0032                    | 30.9309                   | 127.3816                 | 442.2749                 | 1342.5873                | 3656.8435                |
|      | 11 | -3.0856                   | -18.6054                  | -87.8144                 | -343.8868                | -1162.6236               | -3490.8154               |
|      | 12 | 1.4588                    | 10.2908                   | 55.6496                  | 245.7345                 | 925.0521                 | 3061.2053                |
|      | 13 | -0.6386                   | -5.2687                   | -32.6359                 | -162.4649                | -680.8523                | -2482.8122               |
|      | 14 | 0.2603                    | 2.511                     | 17.813                   | 99.949                   | 466.2223                 | 1873.2015                |
|      | 15 | -0.0993                   | -1.1195                   | -9.0931                  | -57.4993                 | -298.4941                | -1321.2085               |
|      | 16 | 0.0356                    | 0.4689                    | 4.3599                   | 31.0654                  | 179.4537                 | 874.9484                 |
|      | 17 | -0.012                    | -0.1852                   | -1.9709                  | -15.8218                 | -101.6921                | -546.0943                |
|      | 18 | 0.0038                    | 0.0692                    | 0.8428                   | 7.6218                   | 54.5                     | 322.3207                 |
| Rb = | 19 | -0.0012                   | -0.0245                   | -0.3419                  | -3.4831                  | -27.7064                 | -180.4459                |
|      | 20 | $3.3667 \times 10^{-4}$   | 0.0083                    | 0.132                    | 1.5141                   | 13.3968                  | 96.0757                  |
|      | 21 | $-9.2703 \times 10^{-5}$  | -0.0027                   | -0.0486                  | -0.6276                  | -6.1761                  | -48.7689                 |
|      | 22 | $2.4398 \times 10^{-5}$   | $8.1789 \times 10^{-4}$   | 0.0171                   | 0.2486                   | 2.7207                   | 23.6534                  |
|      | 23 | $-6.1498 \times 10^{-6}$  | $-2.4078 \times 10^{-4}$  | -0.0058                  | -0.0943                  | -1.1475                  | -10.9834                 |
|      | 24 | $1.4872 \times 10^{-6}$   | $6.8002 \times 10^{-5}$   | 0.0019                   | 0.0343                   | 0.4642                   | 4.8918                   |
|      | 25 | $-3.4565 \times 10^{-7}$  | $-1.8456 \times 10^{-5}$  | $-5.7701 \times 10^{-4}$ | -0.012                   | -0.1805                  | -2.0933                  |
|      | 26 | $7.7322 \times 10^{-8}$   | $4.821 \times 10^{-6}$    | $1.724 \times 10^{-4}$   | 0.004                    | 0.0675                   | 0.862                    |
|      | 27 | $-1.6672 \times 10^{-8}$  | $-1.2138 \times 10^{-6}$  | $-4.9646 \times 10^{-5}$ | -0.0013                  | -0.0243                  | -0.342                   |
|      | 28 | $3.4695 \times 10^{-9}$   | $2.9493 \times 10^{-7}$   | $1.3797 \times 10^{-5}$  | $4.0906 \times 10^{-4}$  | 0.0085                   | 0.131                    |
|      | 29 | $-6.977 \times 10^{-10}$  | $-6.9245 \times 10^{-8}$  | $-3.7047 \times 10^{-6}$ | $-1.2366 \times 10^{-4}$ | -0.0028                  | -0.0484                  |
|      | 30 | $1.3574 \times 10^{-10}$  | $1.5728 \times 10^{-8}$   | $9.6232 \times 10^{-7}$  | $3.6159 \times 10^{-5}$  | $9.2494 \times 10^{-4}$  | 0.0173                   |
|      | 31 | $-2.558 \times 10^{-11}$  | $-3.4596 \times 10^{-9}$  | $-2.4207 \times 10^{-7}$ | $-1.0239 \times 10^{-5}$ | $-2.9118 \times 10^{-4}$ | -0.006                   |
|      | 32 | $4.6597 \times 10^{-12}$  | $7.3772 \times 10^{-10}$  | $5.9028 \times 10^{-8}$  | $2.8104 \times 10^{-6}$  | $8.8856 \times 10^{-5}$  | 0.002                    |
|      | 33 | $-8.1589 \times 10^{-13}$ | $-1.5261 \times 10^{-10}$ | $-1.3966 \times 10^{-8}$ | $-7.4848 \times 10^{-7}$ | $-2.6308 \times 10^{-5}$ | $-6.5734 \times 10^{-4}$ |