

## *Локальный экстремум примеры*

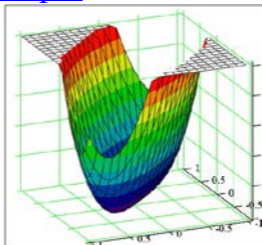
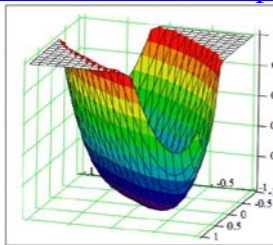
### Многочлен четвертого порядка

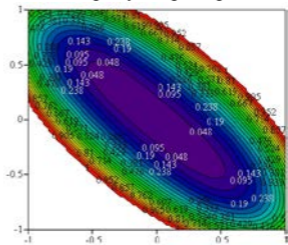
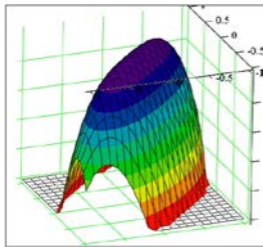
#### Пример 2-1.

$$f(x, y) = x^2 + 2xy + y^2 + x^4 + y^4,$$

[4/m3th05-5c-std1d3plus.mcd](#)

[4/m3th05-5c-std1d3plus.pdf](#)





$$df = \left( 2x + 4x^3 + 2y \quad 2y + 4y^3 + 2x \right) \begin{pmatrix} dx \\ dy \end{pmatrix},$$

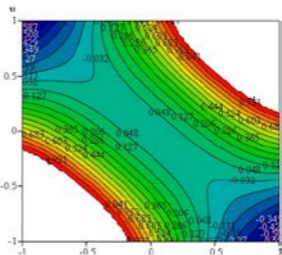
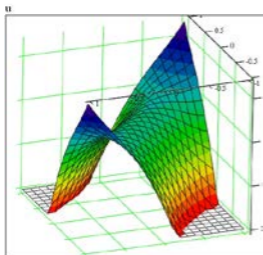
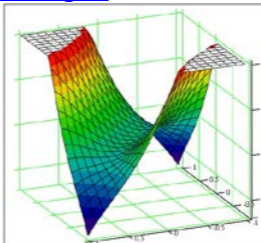
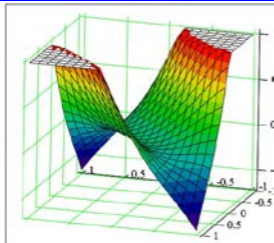
$$M_0 = (0; 0),$$

$$H(0; 0) = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}, \quad A_1 = 2, \quad A_2 = 0.$$

### Пример 2-2.

$$f(x, y) = x^2 + 2xy + y^2 - x^4 - y^4,$$

[4/m3th05-5b-std1d3minus.mcd](#)

[4/m3th05-5b-std1d3minus.pdf](#)

$$df = \begin{pmatrix} 2x - 4x^3 + 2y & 2y - 4y^3 + 2x \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix},$$

$$M_0 = (0; 0),$$

$$H(0; 0) = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}, A_1 = 2, A_2 = 0.$$

### Пример 2-3.

$$f(x, y) = (y - x^2)^2 = x^4 - 2x^2y + y^2,$$

$$df = \begin{pmatrix} 4x^3 - 4xy & 2y - 2x^2 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix},$$

$$M_0 = (0; 0),$$

$$H(x; y) = \begin{pmatrix} 12x^2 - 4y & -4x \\ -4x & 2 \end{pmatrix},$$

$$H(0; 0) = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix},$$

$$A_1 = 2, A_2 = 0.$$

### Пример 2-4.

$$f(x, y) = (y - x^2)^2 + x^4 + y^4,$$

$$df = \begin{pmatrix} 8x^3 - 4xy & 4y^3 + 2y - 2x^2 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix},$$

$$M_0 = (0; 0),$$

$$H(x; y) = \begin{pmatrix} 24x^2 - 4y & -4x \\ -4x & 2 + 12y^2 \end{pmatrix},$$

$$H(0; 0) = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix},$$

$$A_1 = 2, A_2 = 0.$$

### Пример 2-5.

$$f(x, y) = (y - x^2)^2 - (x^4 + y^4),$$

$$df = \left( -4xy \quad -4y^3 + 2y - 2x^2 \right) \begin{pmatrix} dx \\ dy \end{pmatrix},$$

$$M_0 = (0; 0),$$

$$H(x; y) = \begin{pmatrix} -4y & -4x \\ -4x & 2 - 12y^2 \end{pmatrix},$$

$$H(0; 0) = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix},$$

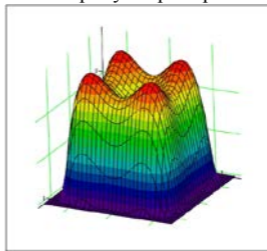
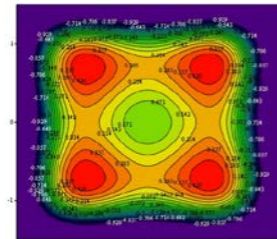
$$A_1 = 2, A_2 = 0.$$

**Пример 5-7.**

$$f(x, y) = 2x^2 + 2y^2 - x^4 - y^4,$$

[4/m3th05-9b-std2.mcd](#)

[4/m3th05-9b-std2.pdf](#)



$$df = \begin{pmatrix} 4x - 4x^3 & 4y - 4y^3 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix}$$

$$H = \begin{pmatrix} 4 - 12x^2 & 0 \\ 0 & 4 - 4y^2 \end{pmatrix}.$$

Условие  $df = 0$  равносильно



$$\begin{cases} 4x(1 - x^2) = 0, \\ 4y(1 - y^2) = 0, \end{cases}$$

девять решений,  $x \in \{-1; 0; 1\}$ ,

$$y \in \{-1; 0; 1\},$$

$$\text{А) } \begin{cases} x = 0, \\ y = 0, \end{cases} \quad H(0; 0) = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix},$$

$$\text{Б) } \begin{cases} x = 1, \\ y = 1, \end{cases} \quad H(1; 1) = \begin{pmatrix} -8 & 0 \\ 0 & -8 \end{pmatrix},$$

$$\text{В) } \begin{cases} x = 1, \\ y = 0, \end{cases} \quad H(1; 0) = \begin{pmatrix} -8 & 0 \\ 0 & 4 \end{pmatrix},$$