

C Alexey A Bykov, 20 oct 2003 boombook@yandex.ru

Collocation method for the electromagnetic scattering problems

A.A.Bykov 2002

$$N := 17 \quad N2 := N \cdot 2 \quad n := 0 \dots N2 \quad j_n := n - N \quad M := N2 + 1$$

$$\lambda := 1.8 \quad T := 1 \quad k_0 := \frac{2 \cdot \pi}{\lambda}$$

$$\varepsilon_1 := 1 \quad \varepsilon_2 := 3.5 \quad k_1 := k_0 \cdot \sqrt{\varepsilon_1} \quad k_2 := k_0 \cdot \sqrt{\varepsilon_2}$$

$$\mu := k_1 \cdot T \cdot \cos(\phi) \quad t_n := \frac{\mu + 2 \cdot \pi \cdot j_n}{T} \quad \gamma_{1n} := \sqrt{k_1^2 - (t_n)^2} \quad \gamma_{2n} := \sqrt{k_2^2 - (t_n)^2}$$

$$\gamma_1^T =$$

	0	1	2	3	4	5	6	7
0	105.011i	98.724i	92.437i	86.149i	79.86i	73.57i	67.279i	60.987i

$$\gamma_2^T =$$

	0	1	2	3	4	5	6	7
0	104.866i	98.57i	92.272i	85.972i	79.669i	73.363i	67.052i	60.736i

$$\alpha := 0.2 \quad f(x) := \alpha \cdot \sin\left(2 \cdot \pi \cdot \frac{x}{T}\right) \quad ff(x) := \frac{d}{dx} f(x)$$

$$K := 2 \cdot N \quad k := 0 \dots K \quad \xi_k := T \cdot \frac{k}{K + 1} \quad \psi_k := f(\xi_k) \quad i := \sqrt{-1}$$

$$nnnx_k := \frac{-ff(\xi_k)}{1 + ff(\xi_k)^2} \quad nnny_k := \frac{1}{1 + ff(\xi_k)^2}$$

$$P1_{k,n} := e^{i \cdot (t_n \cdot \xi_k + \gamma_{1n} \cdot \psi_k)}$$

$$P2_{k,n} := nnnx_k \cdot i \cdot t_n \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma_{1n} \cdot \psi_k)} + nnny_k \cdot i \cdot \gamma_{1n} \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma_{1n} \cdot \psi_k)}$$

$$Q1_{k,n} := -e^{i \cdot (t_n \cdot \xi_k - \gamma_{2n} \cdot \psi_k)}$$

$$Q2_{k,n} := -nnnx_k \cdot i \cdot t_n \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma_{2n} \cdot \psi_k)} + nnny_k \cdot i \cdot \gamma_{2n} \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma_{2n} \cdot \psi_k)}$$

$$B_n := 0 \quad B_N := 1 \quad C_n := 0$$

$$G1_k := -1 \cdot \left[\sum_n e^{i \cdot (t_n \cdot \xi_k - \gamma 1_n \cdot \psi_k)} \cdot B_n \right] + \sum_n e^{i \cdot (t_n \cdot \xi_k + \gamma 2_n \cdot \psi_k)} \cdot C_n$$

$$G2_k := \sum_n \left(-nnnx_k \cdot i \cdot t_n + nnny_k \cdot i \cdot \gamma 1_n \right) \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma 1_n \cdot \psi_k)} \cdot B_n + \sum_n \left(nnnx_k \cdot i \cdot t_n + nnny_k \cdot i \cdot \gamma 2_n \right) \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma 2_n \cdot \psi_k)} \cdot C_n$$

$$AA_{k,n} := P1_{k,n} \qquad AA_{k,M+n} := Q1_{k,n} \qquad AA_{M+k,n} := P2_{k,n} \qquad AA_{M+k,M+n} := Q2_{k,n}$$

$$BB_k := G1_k \quad BB_{M+k} := G2_k$$

$$G1^T =$$

	0	1	2	3
0	-1	-0.998+0.058i	-0.994+0.112i	-0.987+0.16i

$$G2^T =$$

	0	1	2	3
0	2.022i	0.119+2.045i	0.239+2.116i	0.362+2.238i

$$BB^T =$$

	0	1	2	3
0	-1	-0.998+0.058i	-0.994+0.112i	-0.987+0.16i

$$Sol := AA^{-1} \cdot BB$$

$$Sol^T =$$

	0	1	2
0	2.723·10 ⁻¹⁰ +1.675i·10 ⁻⁸	4.424·10 ⁻⁷ -1.098i·10 ⁻⁸	-1.959·10 ⁻⁷ -5.461i·10 ⁻⁶

$$A_n := Sol_n \qquad D_n := Sol_{M+n}$$

$$A^T =$$

	0	1	2
0	2.723·10 ⁻¹⁰ +1.675i·10 ⁻⁸	4.424·10 ⁻⁷ -1.098i·10 ⁻⁸	-1.959·10 ⁻⁷ -5.461i·10 ⁻⁶

$$D^T =$$

	0	1	2
0	-1.661·10 ⁻⁸ +1.666i·10 ⁻⁸	-4.316·10 ⁻⁷ -4.229i·10 ⁻⁷	4.986·10 ⁻⁶ -5.206i·10 ⁻⁶

$$ma_{n,0} := \left| A_n \right| \qquad md_{n,0} := \left| D_n \right| \qquad ma_{n,1} := \gamma 1_n \qquad md_{n,1} := \gamma 2_n$$

$$\phi \equiv \frac{60}{180} \cdot \pi$$

$\mathbf{ma}^T =$

	0	1	2	3	4	5	6	7
0	$1.675 \cdot 10^{-8}$	$4.425 \cdot 10^{-7}$	$5.464 \cdot 10^{-6}$	$4.193 \cdot 10^{-5}$	$2.244 \cdot 10^{-4}$	$8.922 \cdot 10^{-4}$	$2.745 \cdot 10^{-3}$	$6.737 \cdot 10^{-3}$
1	105.011i	98.724i	92.437i	86.149i	79.86i	73.57i	67.279i	60.987i

$\mathbf{md}^T =$

	0	1	2	3	4	5	6	7
0	$2.353 \cdot 10^{-8}$	$6.043 \cdot 10^{-7}$	$7.208 \cdot 10^{-6}$	$5.299 \cdot 10^{-5}$	$2.689 \cdot 10^{-4}$	$1 \cdot 10^{-3}$	$2.833 \cdot 10^{-3}$	$6.283 \cdot 10^{-3}$
1	104.866i	98.57i	92.272i	85.972i	79.669i	73.363i	67.052i	60.736i

$$\sum_n \operatorname{Re}(\gamma_{1n}) \cdot \left(\left|A_n\right|\right)^2 + \sum_n \operatorname{Re}(\gamma_{2n}) \cdot \left(\left|D_n\right|\right)^2 = 3.023$$

$$\sum_n \operatorname{Re}(\gamma_{1n}) \cdot \left(\left|B_n\right|\right)^2 + \sum_n \operatorname{Re}(\gamma_{2n}) \cdot \left(\left|C_n\right|\right)^2 = 3.023$$

$$u1(x,y) := \sum_n \left[A_n \cdot e^{i \cdot \left(t_n \cdot x + \gamma_{1n} \cdot y\right)} + B_n \cdot e^{i \cdot \left(t_n \cdot x - \gamma_{1n} \cdot y\right)} \right]$$

$$u2(x,y) := \sum_n \left[C_n \cdot e^{i \cdot \left(t_n \cdot x + \gamma_{2n} \cdot y\right)} + D_n \cdot e^{i \cdot \left(t_n \cdot x - \gamma_{2n} \cdot y\right)} \right]$$

$$u(x,y) := \left\{ \begin{array}{l} u1(x,y) \text{ if } y > f(x) \\ u2(x,y) \text{ otherwise} \end{array} \right.$$

$$N_x := 125 \qquad N_y := 125 \qquad n_x := 0 \ldots N_x \qquad n_y := 0 \ldots N_y \qquad b := 3$$

$$xx_{n_x} := b \cdot \frac{n_x}{N_x} \qquad yy_{n_y} := -b + 2 \cdot b \cdot \frac{n_y}{N_y}$$

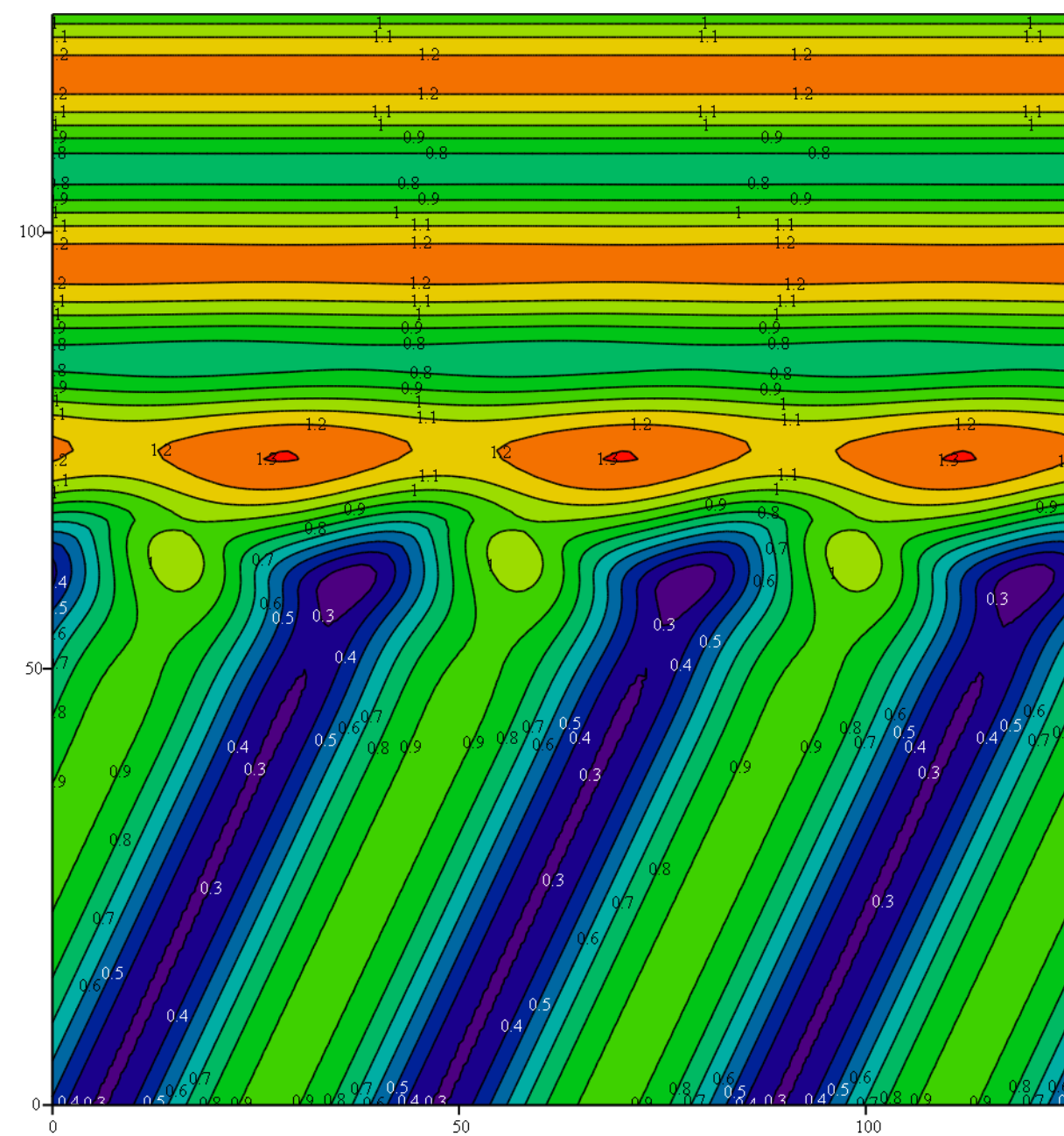
$\mathbf{xx}^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.024	0.048	0.072	0.096	0.12	0.144	0.168	0.192	0.216

$\mathbf{yy}^T =$

	0	1	2	3	4	5	6	7	8
0	-3	-2.952	-2.904	-2.856	-2.808	-2.76	-2.712	-2.664	-2.616

$$U_{n_y,n_x} := u\Bigl(xx_{n_x},yy_{n_y}\Bigr) \qquad V_{n_y,n_x} := \left|U_{n_y,n_x}\right|$$



v^T