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Collocation method for the electromagnetic scatteering problems

A.A.Bykov 2002

$N := 17 \quad N2 := N \cdot 2 \quad n := 0 .. N2 \quad j_n := n - N \quad M := N2 + 1$

$\lambda := 1.8 \quad T := 1 \quad k_0 := \frac{2 \cdot \pi}{\lambda}$

$\varepsilon_1 := 1 \quad \varepsilon_2 := 7.5 \quad k_1 := k_0 \cdot \sqrt{\varepsilon_1} \quad k_2 := k_0 \cdot \sqrt{\varepsilon_2}$

$\mu := k_1 \cdot T \cdot \cos(\phi) \quad t_n := \frac{\mu + 2 \cdot \pi \cdot j_n}{T} \quad \gamma 1_n := \sqrt{k_1^2 - (t_n)^2} \quad \gamma 2_n := \sqrt{k_2^2 - (t_n)^2}$

$\gamma 1^T =$

	0	1	2	3	4	5	6	7
0	105.011i	98.724i	92.437i	86.149i	79.86i	73.57i	67.279i	60.987i

$\gamma 2^T =$

	0	1	2	3	4	5	6	7
0	104.633i	98.322i	92.007i	85.688i	79.362i	73.03i	66.688i	60.334i

$\alpha := 0.22 \quad f(x) := \alpha \cdot \sin\left(2 \cdot \pi \cdot \frac{x}{T}\right) \quad ff(x) := \frac{d}{dx} f(x)$

$K := 2 \cdot N \quad k := 0 .. K \quad \xi_k := T \cdot \frac{k}{K + 1} \quad \psi_k := f(\xi_k) \quad i := \sqrt{-1}$

$nnnx_k := \frac{-ff(\xi_k)}{1 + ff(\xi_k)^2} \quad nnny_k := \frac{1}{1 + ff(\xi_k)^2}$

$P1_{k,n} := e^{i \cdot (t_n \cdot \xi_k + \gamma 1_n \cdot \psi_k)}$

$P2_{k,n} := nnnx_k \cdot i \cdot t_n \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma 1_n \cdot \psi_k)} + nnny_k \cdot i \cdot \gamma 1_n \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma 1_n \cdot \psi_k)}$

$Q1_{k,n} := -e^{i \cdot (t_n \cdot \xi_k - \gamma 2_n \cdot \psi_k)}$

$Q2_{k,n} := -nnnx_k \cdot i \cdot t_n \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma 2_n \cdot \psi_k)} + nnny_k \cdot i \cdot \gamma 2_n \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma 2_n \cdot \psi_k)}$

$B_n := 0 \quad B_N := 1 \quad C_n := 0$

$$G1_k := -1\cdot\left[\sum_n e^{i\cdot\left(t_n\cdot\xi_k-\gamma1_n\cdot\psi_k\right)}\cdot B_n\right] + \sum_n e^{i\cdot\left(t_n\cdot\xi_k+\gamma2_n\cdot\psi_k\right)}\cdot C_n$$

$$G2_k := \sum_n \left(-nnnx_k\cdot i\cdot t_n + nnny_k\cdot i\cdot\gamma1_n\right)\cdot e^{i\cdot\left(t_n\cdot\xi_k-\gamma1_n\cdot\psi_k\right)}\cdot B_n + \sum_n \left(nnnx_k\cdot i\cdot t_n + nnny_k\cdot i\cdot\gamma2_n\right)\cdot e^{i\cdot\left(t_n\cdot\xi_k+\gamma2_n\cdot\psi_k\right)}\cdot C_n$$

$AA_{k,n} := P1_{k,n}$ $\quad AA_{k,M+n} := Q1_{k,n}$ $\quad AA_{M+k,n} := P2_{k,n}$ $\quad AA_{M+k,M+n} := Q2_{k,n}$

$BB_k := G1_k$ $\quad BB_{M+k} := G2_k$

$G1^T =$

	0	1	2	3
0	-1	-0.998+0.069i	-0.991+0.134i	-0.982+0.19i

$G2^T =$

	0	1	2
0	1.867i	0.13+1.889i	0.264+1.957i

$BB^T =$

	0	1	2
0	-1	-0.998+0.069i	-0.991+0.134i

$Sol := AA^{-1}\cdot BB$

$Sol^T =$

	0	1	2
0	$-2.241\cdot10^{-9}+5.754i\cdot10^{-9}$	$1.921\cdot10^{-7}+7.749i\cdot10^{-8}$	$1.235\cdot10^{-6}-3.002i\cdot10^{-6}$

$A_n := Sol_n$ $\qquad D_n := Sol_{M+n}$

$A^T =$

	0	1	2
0	$-2.241\cdot10^{-9}+5.754i\cdot10^{-9}$	$1.921\cdot10^{-7}+7.749i\cdot10^{-8}$	$1.235\cdot10^{-6}-3.002i\cdot10^{-6}$

$D^T =$

	0	1	2
0	$-1.395\cdot10^{-8}+1.258i\cdot10^{-8}$	$-3.814\cdot10^{-7}-4.191i\cdot10^{-7}$	$5.865\cdot10^{-6}-5.403i\cdot10^{-6}$

$ma_{n,0} := \left|A_n\right|$ $\quad md_{n,0} := \left|D_n\right|$ $\quad ma_{n,1} := \gamma1_n$ $\quad md_{n,1} := \gamma2_n$

$$\phi \equiv \frac{60}{180}\cdot\pi$$

$ma^T =$

	0	1	2	3	4	5	6	7
0	$6.175\cdot10^{-9}$	$2.072\cdot10^{-7}$	$3.246\cdot10^{-6}$	$3.151\cdot10^{-5}$	$2.12\cdot10^{-4}$	$1.047\cdot10^{-3}$	$3.935\cdot10^{-3}$	0.012
1	105.011i	98.724i	92.437i	86.149i	79.86i	73.57i	67.279i	60.987i

$md^T =$

	0	1	2	3	4	5	6	7
0	$1.878\cdot10^{-8}$	$5.667\cdot10^{-7}$	$7.974\cdot10^{-6}$	$6.956\cdot10^{-5}$	$4.216\cdot10^{-4}$	$1.887\cdot10^{-3}$	$6.471\cdot10^{-3}$	0.017
1	104.633i	98.322i	92.007i	85.688i	79.362i	73.03i	66.688i	60.334i

$$\sum_n \mathrm{Re}(\gamma 1_n)\cdot\left(\left|A_n\right|\right)^2 + \sum_n \mathrm{Re}(\gamma 2_n)\cdot\left(\left|D_n\right|\right)^2 = 3.019$$

$$\sum_n \mathrm{Re}(\gamma 1_n)\cdot\left(\left|B_n\right|\right)^2 + \sum_n \mathrm{Re}(\gamma 2_n)\cdot\left(\left|C_n\right|\right)^2 = 3.023$$

$$u1(x,y) := \sum_n \left[A_n\cdot e^{i\cdot(t_n\cdot x+\gamma 1_n\cdot y)} + B_n\cdot e^{i\cdot(t_n\cdot x-\gamma 1_n\cdot y)}\right]$$

$$u2(x,y) := \sum_n \left[C_n\cdot e^{i\cdot(t_n\cdot x+\gamma 2_n\cdot y)} + D_n\cdot e^{i\cdot(t_n\cdot x-\gamma 2_n\cdot y)}\right]$$

$$u(x,y) := \begin{vmatrix} u1(x,y) & \text{if } y > f(x) \\ u2(x,y) & \text{otherwise} \end{vmatrix}$$

$Nx := 125 \qquad Ny := 125 \qquad nx := 0..Nx \qquad ny := 0..Ny \qquad b := 3$

$$xx_{nx} := b\cdot\frac{nx}{Nx} \qquad yy_{ny} := -b + 2\cdot b\cdot\frac{ny}{Ny}$$

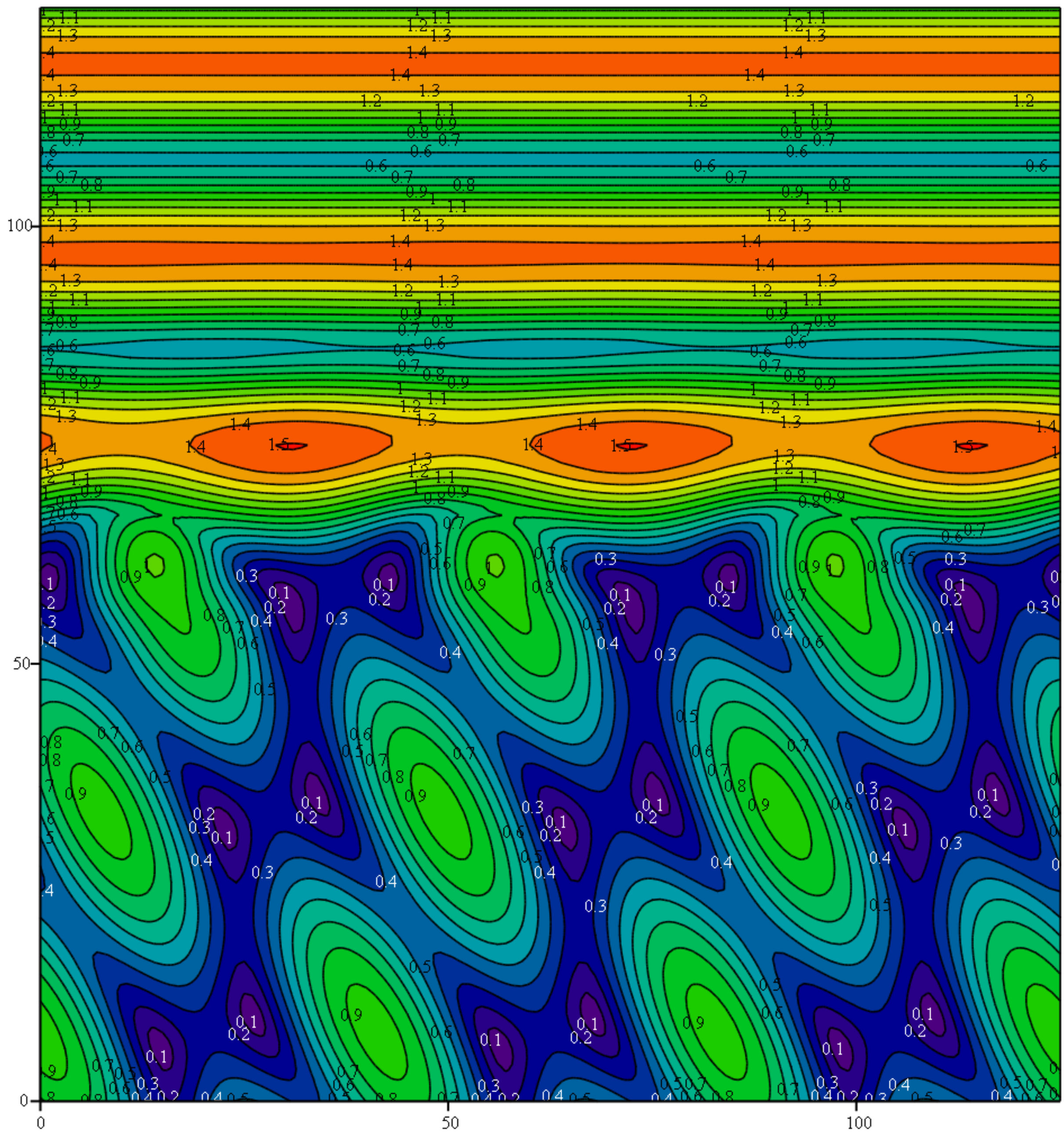
$xx^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.024	0.048	0.072	0.096	0.12	0.144	0.168	0.192	0.216

$yy^T =$

	0	1	2	3	4	5	6	7	8
0	-3	-2.952	-2.904	-2.856	-2.808	-2.76	-2.712	-2.664	-2.616

$$U_{ny,nx} := u(xx_{nx}, yy_{ny}) \qquad V_{ny,nx} := \left|U_{ny,nx}\right|$$



V^T