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Collocation method for the electromagnetic scatteering problems

A.A.Bykov 2002

$N := 27 \quad N2 := N \cdot 2 \quad n := 0 .. N2 \quad j_n := n - N \quad M := N2 + 1$

$\lambda := 0.8 \quad T := 1 \quad k_0 := \frac{2 \cdot \pi}{\lambda}$

$\varepsilon_1 := 1 \quad \varepsilon_2 := 7.5 \quad k_1 := k_0 \cdot \sqrt{\varepsilon_1} \quad k_2 := k_0 \cdot \sqrt{\varepsilon_2} \quad \phi \equiv \frac{60}{180} \cdot \pi$

$\mu := k_1 \cdot T \cdot \sin(\phi) \quad t_n := \frac{\mu + 2 \cdot \pi \cdot j_n}{T} \quad \gamma 1_n := \sqrt{k_1^2 - (t_n)^2} \quad \gamma 2_n := \sqrt{k_2^2 - (t_n)^2}$

$\gamma 1^T =$

	3	4	5	6	7	8
0	143.78i	137.487i	131.193i	124.898i	118.602i	112.304i

$\gamma 2^T =$

	0	1	2	3	4	5
0	161.418i	155.077i	148.731i	142.379i	136.021i	129.656i

$\alpha := 0.0008 \quad f(x) := \alpha \cdot \cos\left(2 \cdot \pi \cdot \frac{x}{T}\right) \quad ff(x) := \frac{d}{dx} f(x)$

$K := 2 \cdot N \quad k := 0 .. K \quad \xi_k := T \cdot \frac{k}{K + 1} \quad \psi_k := f(\xi_k) \quad i := \sqrt{-1}$

$nnnx_k := \frac{-ff(\xi_k)}{1 + ff(\xi_k)^2} \quad nnny_k := \frac{1}{1 + ff(\xi_k)^2}$

$P1_{k,n} := e^{i \cdot (t_n \cdot \xi_k + \gamma 1_n \cdot \psi_k)}$

$P2_{k,n} := nnnx_k \cdot i \cdot t_n \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma 1_n \cdot \psi_k)} + nnny_k \cdot i \cdot \gamma 1_n \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma 1_n \cdot \psi_k)}$

$Q1_{k,n} := -e^{i \cdot (t_n \cdot \xi_k - \gamma 2_n \cdot \psi_k)}$

$Q2_{k,n} := -nnnx_k \cdot i \cdot t_n \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma 2_n \cdot \psi_k)} + nnny_k \cdot i \cdot \gamma 2_n \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma 2_n \cdot \psi_k)}$

$B_n := 0 \quad B_N := 1 \quad C_n := 0$

$$G1_k := -1 \cdot \left[\sum_n e^{i \cdot \left(t_n \cdot \xi_k - \gamma 1_n \cdot \psi_k \right)} \cdot B_n \right] + \sum_n e^{i \cdot \left(t_n \cdot \xi_k + \gamma 2_n \cdot \psi_k \right)} \cdot C_n$$

$$G2_k := \sum_n \left(-nnnx_k \cdot i \cdot t_n + nnny_k \cdot i \cdot \gamma 1_n \right) \cdot e^{i \cdot \left(t_n \cdot \xi_k - \gamma 1_n \cdot \psi_k \right)} \cdot B_n + \sum_n \left(nnnx_k \cdot i \cdot t_n + nnny_k \cdot i \cdot \gamma 2_n \right) \cdot e^{i \cdot \left(t_n \cdot \xi_k + \gamma 2_n \cdot \psi_k \right)} \cdot C_n$$

$AA_{k,n} := P1_{k,n}$ $xAA_{k,M+n} := Q1_{k,n}$ $xAA_{M+k,n} := P2_{k,n}$ $xAA_{M+k,M+n} := Q2_{k,n}$

$BB_k := G1_k$ $xBB_{M+k} := G2_k$

$G1^T =$

	0	1	2	3
0	-1+3.142i·10 $^{-3}$	-0.993-0.12i	-0.97-0.242i	-0.933-0.36i

$G2^T =$

	0	1	2	3
0	0.012+3.927i	-0.472+3.895i	-0.948+3.803i	-1.409+3.653i

$BB^T =$

	0	1	2	3
0	-1+3.142i·10 $^{-3}$	-0.993-0.12i	-0.97-0.242i	-0.933-0.36i

$Sol := AA^{-1} \cdot BB$

$Sol^T =$

	0	1	2
0	0	0	0

$A_n := Sol_n$ $D_n := 0 \cdot Sol_{0M+n}$

$A^T =$

	0	1	2
0	0	0	0

$D^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0

$ma_{n,0} := |A_n|$ $md_{n,0} := |D_n|$ $ma_{n,1} := \gamma 1_n$ $md_{n,1} := \gamma 2_n$

$ma^T =$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	162.655i	156.364i	150.073i	143.78i	137.487i	131.193i

$md^T =$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	161.418i	155.077i	148.731i	142.379i	136.021i	129.656i

$$\sum_n \mathrm{Re}(\gamma 1_n)\cdot\left(\left|A_n\right|\right)^2 + \sum_n \mathrm{Re}(\gamma 2_n)\cdot\left(\left|D_n\right|\right)^2 = 3.927$$

$$\sum_n \mathrm{Re}(\gamma 1_n)\cdot\left(\left|B_n\right|\right)^2 + \sum_n \mathrm{Re}(\gamma 2_n)\cdot\left(\left|C_n\right|\right)^2 = 3.927$$

$$u1(x,y) := \sum_n \left[A_n \cdot e^{i\cdot\left(t_n\cdot x + \gamma 1_n\cdot y\right)} + B_n \cdot e^{i\cdot\left(t_n\cdot x - \gamma 1_n\cdot y\right)} \right]$$

$$u2(x,y) := \sum_n \left[C_n \cdot e^{i\cdot\left(t_n\cdot x + \gamma 2_n\cdot y\right)} + D_n \cdot e^{i\cdot\left(t_n\cdot x - \gamma 2_n\cdot y\right)} \right]$$

$$u(x,y) := \begin{cases} u1(x,y) & \text{if } y > f(x) \\ u2(x,y) & \text{otherwise} \end{cases}$$

$Nx := 125 \qquad Ny := 125 \qquad nx := 0 \,..\, Nx \qquad ny := 0 \,..\, Ny \qquad b := \frac{T}{2}$

$$xx_{nx} := 3T\cdot\frac{nx}{Nx} \qquad yy_{ny} := -b + 7\cdot b\cdot\frac{ny}{Ny}$$

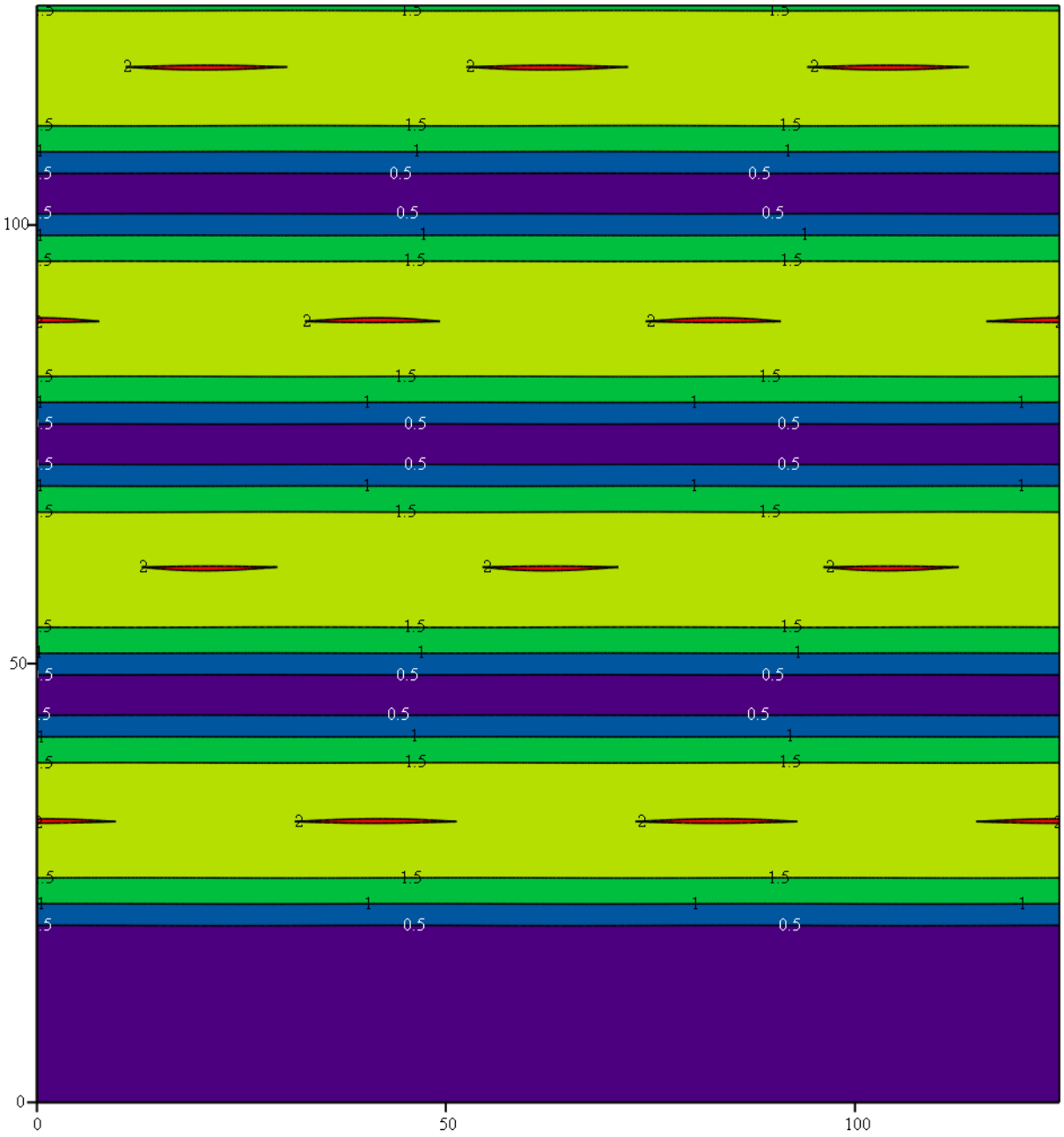
$xx^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.024	0.048	0.072	0.096	0.12	0.144	0.168	0.192	0.216

$yy^T =$

	0	1	2	3	4	5	6	7	8
0	-0.5	-0.472	-0.444	-0.416	-0.388	-0.36	-0.332	-0.304	-0.276

$$U_{ny,nx} := u\left(xx_{nx}, yy_{ny}\right) \qquad V_{ny,nx} := \left|U_{ny,nx}\right|$$



V^T