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Collocation method for the electromagnetic scatteering problems

A.A.Bykov 2002

$N := 17 \quad N2 := N\cdot 2 \quad n := 0 .. N2 \quad j_n := n - N \quad M := N2 + 1$

$\lambda := 0.8 \quad T := 1 \quad k_0 := \dfrac{2\cdot\pi}{\lambda}$

$\varepsilon_1 := 1 \quad \varepsilon_2 := 7.5 \quad k_1 := k_0\cdot\sqrt{\varepsilon_1} \quad k_2 := k_0\cdot\sqrt{\varepsilon_2} \quad \phi \equiv \dfrac{0}{180}\cdot\pi$

$\mu := k_1\cdot T\cdot\sin(\phi) \quad t_n := \dfrac{\mu + 2\cdot\pi\cdot j_n}{T} \quad \gamma1_n := \sqrt{{k_1}^2 - (t_n)^2} \quad \gamma2_n := \sqrt{{k_2}^2 - (t_n)^2}$

$\gamma1^T =$

	3	4	5	6	7	8
0	87.613i	81.303i	74.988i	68.667i	62.339i	56.001i

$\gamma2^T =$

	0	1	2	3	4	5
0	104.626i	98.203i	91.761i	85.294i	78.799i	72.265i

$\alpha := 0.095 \quad f(x) := \alpha\cdot\cos\left(2\cdot\pi\cdot\dfrac{x}{T}\right) \quad ff(x) := \dfrac{d}{dx}f(x)$

$K := 2\cdot N \quad k := 0 .. K \quad \xi_k := T\cdot\dfrac{k}{K+1} \quad \psi_k := f(\xi_k) \quad i := \sqrt{-1}$

$nnnx_k := \dfrac{-ff(\xi_k)}{1 + ff(\xi_k)^2} \quad nnny_k := \dfrac{1}{1 + ff(\xi_k)^2}$

$P1_{k,n} := e^{i\cdot(t_n\cdot\xi_k + \gamma1_n\cdot\psi_k)}$

$P2_{k,n} := nnnx_k\cdot i\cdot t_n\cdot e^{i\cdot(t_n\cdot\xi_k + \gamma1_n\cdot\psi_k)} + nnny_k\cdot i\cdot\gamma1_n\cdot e^{i\cdot(t_n\cdot\xi_k + \gamma1_n\cdot\psi_k)}$

$Q1_{k,n} := -e^{i\cdot(t_n\cdot\xi_k - \gamma2_n\cdot\psi_k)}$

$Q2_{k,n} := -nnnx_k\cdot i\cdot t_n\cdot e^{i\cdot(t_n\cdot\xi_k - \gamma2_n\cdot\psi_k)} + nnny_k\cdot i\cdot\gamma2_n\cdot e^{i\cdot(t_n\cdot\xi_k - \gamma2_n\cdot\psi_k)}$

$B_n := 0 \quad B_N := 1 \quad C_n := 0$

$$G1_k := -1\cdot\left[\sum_n e^{i\cdot(t_n\cdot\xi_k - \gamma1_n\cdot\psi_k)}\cdot B_n\right] + \sum_n e^{i\cdot(t_n\cdot\xi_k + \gamma2_n\cdot\psi_k)}\cdot C_n$$

$$G2_k := \sum_n \left(-nnnx_k\cdot i\cdot t_n + nnny_k\cdot i\cdot\gamma1_n\right)\cdot e^{i\cdot(t_n\cdot\xi_k - \gamma1_n\cdot\psi_k)}\cdot B_n + \sum_n \left(nnnx_k\cdot i\cdot t_n + nnny_k\cdot i\cdot\gamma2_n\right)\cdot e^{i\cdot(t_n\cdot\xi_k + \gamma2_n\cdot\psi_k)}\cdot C_n$$

$AA_{k,n} := P1_{k,n}$ $\quad xAA_{k,M+n} := Q1_{k,n}$ $\quad xAA_{M+k,n} := P2_{k,n}$ $\quad xAA_{M+k,M+n} := Q2_{k,n}$

$BB_k := G1_k$ $\quad xBB_{M+k} := G2_k$

$G1^T =$

	0	1	2	3
0	-0.734+0.679i	-0.742+0.67i	-0.766+0.643i	-0.802+0.598i

$G2^T =$

	0	1	2	3	4
0	5.331+5.767i	5.203+5.765i	4.838+5.761i	4.291+5.758i	3.625+5.758i

$BB^T =$

	0	1	2	3
0	-0.734+0.679i	-0.742+0.67i	-0.766+0.643i	-0.802+0.598i

$Sol := AA^{-1}\cdot BB$

$Sol^T =$

	0	1	2
0	$2.265\cdot10^{-3}$-$5.041i\cdot10^{-5}$	$2.525\cdot10^{-3}$-$5.829i\cdot10^{-5}$	$2.594\cdot10^{-3}$-$6.444i\cdot10^{-5}$

$A_n := Sol_n$ $\quad D_n := 0\cdot Sol_{0M+n}$

$A^T =$

	0	1	2
0	$2.265\cdot10^{-3}$-$5.041i\cdot10^{-5}$	$2.525\cdot10^{-3}$-$5.829i\cdot10^{-5}$	$2.594\cdot10^{-3}$-$6.444i\cdot10^{-5}$

$D^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0

$ma_{n,0} := |A_n|$ $\quad md_{n,0} := |D_n|$ $\quad ma_{n,1} := \gamma1_n$ $\quad md_{n,1} := \gamma2_n$

$ma^T =$

	0	1	2	3	4	5
0	$2.265\cdot10^{-3}$	$2.525\cdot10^{-3}$	$2.594\cdot10^{-3}$	$3.267\cdot10^{-3}$	$4.057\cdot10^{-3}$	$5.155\cdot10^{-3}$
1	106.525i	100.224i	93.92i	87.613i	81.303i	74.988i

$md^T =$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	104.626i	98.203i	91.761i	85.294i	78.799i	72.265i

$$\sum_n \mathrm{Re}(\gamma1_n)\cdot\left(\left|A_n\right|\right)^2 + \sum_n \mathrm{Re}(\gamma2_n)\cdot\left(\left|D_n\right|\right)^2 = 7.854$$

$$\sum_n \mathrm{Re}(\gamma1_n)\cdot\left(\left|B_n\right|\right)^2 + \sum_n \mathrm{Re}(\gamma2_n)\cdot\left(\left|C_n\right|\right)^2 = 7.854$$

$$u1(x,y) := \sum_n \left[A_n\cdot e^{i\cdot(t_n\cdot x+\gamma1_n\cdot y)} + B_n\cdot e^{i\cdot(t_n\cdot x-\gamma1_n\cdot y)}\right]$$

$$u2(x,y) := \sum_n \left[C_n\cdot e^{i\cdot(t_n\cdot x+\gamma2_n\cdot y)} + D_n\cdot e^{i\cdot(t_n\cdot x-\gamma2_n\cdot y)}\right]$$

$$u(x,y) := \left| \begin{array}{ll} u1(x,y) & \text{if } y > f(x) \\ u2(x,y) & \text{otherwise} \end{array} \right.$$

$Nx := 125 \qquad Ny := 125 \qquad nx := 0..Nx \qquad ny := 0..Ny \qquad b := \frac{T}{2}$

$xx_{nx} := 3T\cdot\frac{nx}{Nx} \qquad yy_{ny} := -b + 7\cdot b\cdot\frac{ny}{Ny}$

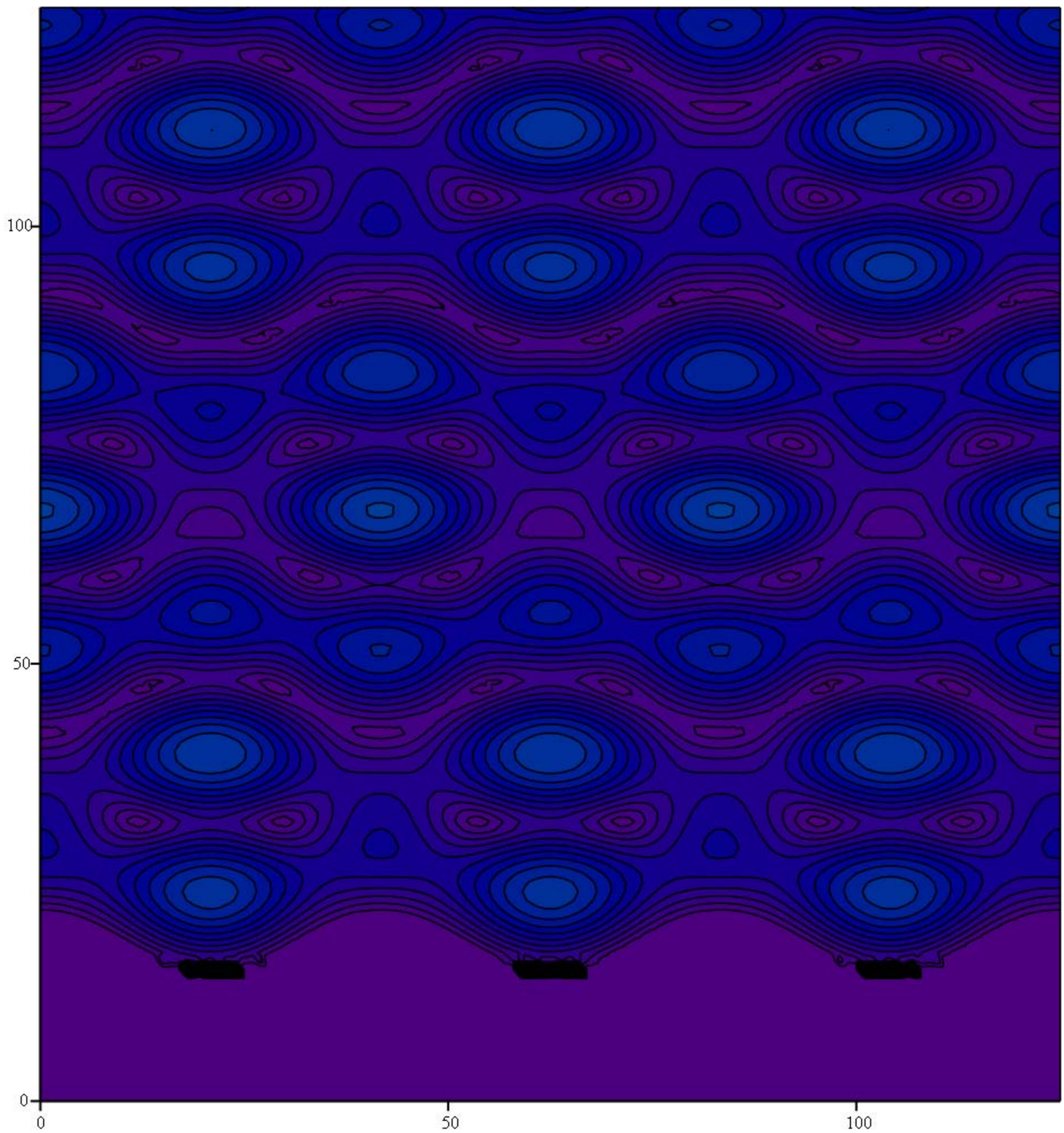
$xx^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.024	0.048	0.072	0.096	0.12	0.144	0.168	0.192	0.216

$yy^T =$

	0	1	2	3	4	5	6	7	8
0	-0.5	-0.472	-0.444	-0.416	-0.388	-0.36	-0.332	-0.304	-0.276

$U_{ny,nx} := u\left(xx_{nx}, yy_{ny}\right) \qquad V_{ny,nx} := \left|U_{ny,nx}\right|$



V^T