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Collocation method for the electromagnetic scatteering problems

A.A.Bykov 2002

$N := 17 \qquad N2 := N \cdot 2 \qquad n := 0 .. N2 \qquad j_n := n - N \qquad M := N2 + 1$

$\lambda := 0.8 \qquad T := 1 \qquad k_0 := \frac{2 \cdot \pi}{\lambda}$

$\varepsilon_1 := 1 \qquad \varepsilon_2 := 7.5 \qquad k_1 := k_0 \cdot \sqrt{\varepsilon_1} \quad k_2 := k_0 \cdot \sqrt{\varepsilon_2} \qquad \phi \equiv \frac{60}{180} \cdot \pi$

$\mu := k_1 \cdot T \cdot \sin(\phi) \qquad t_n := \frac{\mu + 2 \cdot \pi \cdot j_n}{T} \qquad \gamma1_n := \sqrt{{k_1}^2 - (t_n)^2} \qquad \gamma2_n := \sqrt{{k_2}^2 - (t_n)^2}$

$\gamma1^T =$

	3	4	5	6	7	8
0	80.782i	74.467i	68.145i	61.816i	55.477i	49.123i

$\gamma2^T =$

	0	1	2	3	4	5
0	97.672i	91.228i	84.759i	78.261i	71.724i	65.137i

$\alpha := 0.08 \qquad f(x) := \alpha \cdot \left(\cos\left(2 \cdot \pi \cdot \frac{x}{T}\right) + 0 \cdot \cos\left(4 \cdot \pi \cdot \frac{x}{T}\right) \right) \qquad ff(x) := \frac{d}{dx} f(x)$

$K := 2 \cdot N \qquad k := 0 .. K \qquad \xi_k := T \cdot \frac{k}{K+1} \quad \psi_k := f(\xi_k) \qquad i := \sqrt{-1}$

$nnnx_k := \frac{-ff(\xi_k)}{1 + ff(\xi_k)^2} \qquad nnny_k := \frac{1}{1 + ff(\xi_k)^2}$

$P1_{k,n} := e^{i \cdot (t_n \cdot \xi_k + \gamma1_n \cdot \psi_k)}$

$P2_{k,n} := nnnx_k \cdot i \cdot t_n \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma1_n \cdot \psi_k)} + nnny_k \cdot i \cdot \gamma1_n \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma1_n \cdot \psi_k)}$

$Q1_{k,n} := -e^{i \cdot (t_n \cdot \xi_k - \gamma2_n \cdot \psi_k)}$

$Q2_{k,n} := -nnnx_k \cdot i \cdot t_n \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma2_n \cdot \psi_k)} + nnny_k \cdot i \cdot \gamma2_n \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma2_n \cdot \psi_k)}$

$B_n := 0 \qquad B_N := 1 \qquad C_n := 0$

$$G1_k := -1\cdot\left[\sum_n e^{i\cdot(t_n\cdot\xi_k-\gamma 1_n\cdot\psi_k)}\cdot B_n\right] + \sum_n e^{i\cdot(t_n\cdot\xi_k+\gamma 2_n\cdot\psi_k)}\cdot C_n$$

$$G2_k := \sum_n \left(-nnnx_k\cdot i\cdot t_n + nnny_k\cdot i\cdot\gamma 1_n\right)\cdot e^{i\cdot(t_n\cdot\xi_k-\gamma 1_n\cdot\psi_k)}\cdot B_n + \sum_n \left(nnnx_k\cdot i\cdot t_n + nnny_k\cdot i\cdot\gamma 2_n\right)\cdot e^{i\cdot(t_n\cdot\xi_k+\gamma 2_n\cdot\psi_k)}\cdot C_n$$

$AA_{k,n} := P1_{k,n}$ $\quad xAA_{k,M+n} := Q1_{k,n}$ $\quad xAA_{M+k,n} := P2_{k,n}$ $\quad xAA_{M+k,M+n} := Q2_{k,n}$

$BB_k := G1_k$ $\quad xBB_{M+k} := G2_k$

$G1^T =$

	0	1	2	3
0	-0.951+0.309i	-0.993+0.115i	-0.996-0.094i	-0.951-0.308i

$G2^T =$

	0	1	2	3
0	1.214+3.735i	0.377+3.268i	-0.25+2.631i	-0.628+1.939i

$BB^T =$

	0	1	2	3
0	-0.951+0.309i	-0.993+0.115i	-0.996-0.094i	-0.951-0.308i

$Sol := AA^{-1}\cdot BB$

$Sol^T =$

	0	1	2
0	$7.267\cdot 10^{-6}+5.551i\cdot 10^{-5}$	$3.719\cdot 10^{-6}-1.819i\cdot 10^{-5}$	$7.5\cdot 10^{-6}-3.793i\cdot 10^{-6}$

$A_n := Sol_n$ $\qquad D_n := 0\cdot Sol_{0M+n}$

$A^T =$

	0	1	2
0	$7.267\cdot 10^{-6}+5.551i\cdot 10^{-5}$	$3.719\cdot 10^{-6}-1.819i\cdot 10^{-5}$	$7.5\cdot 10^{-6}-3.793i\cdot 10^{-6}$

$D^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0

$ma_{n,0} := \left|A_n\right|$ $\quad md_{n,0} := \left|D_n\right|$ $\quad ma_{n,1} := \gamma 1_n$ $\quad md_{n,1} := \gamma 2_n$

$ma^T =$

	0	1	2	3	4	5
0	$5.599\cdot10^{-5}$	$1.856\cdot10^{-5}$	$8.404\cdot10^{-6}$	$2.201\cdot10^{-5}$	$2.79\cdot10^{-5}$	$4.64\cdot10^{-5}$
1	99.704i	93.4i	87.093i	80.782i	74.467i	68.145i

$md^T =$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	97.672i	91.228i	84.759i	78.261i	71.724i	65.137i

$$\sum_n \mathrm{Re}(\gamma1_n)\cdot\left(\left|A_n\right|\right)^2 + \sum_n \mathrm{Re}(\gamma2_n)\cdot\left(\left|D_n\right|\right)^2 = 3.927$$

$$\sum_n \mathrm{Re}(\gamma1_n)\cdot\left(\left|B_n\right|\right)^2 + \sum_n \mathrm{Re}(\gamma2_n)\cdot\left(\left|C_n\right|\right)^2 = 3.927$$

$$u1(x,y) := \sum_n \left[A_n\cdot e^{i\cdot(t_n\cdot x+\gamma1_n\cdot y)} + B_n\cdot e^{i\cdot(t_n\cdot x-\gamma1_n\cdot y)}\right]$$

$$u2(x,y) := \sum_n \left[C_n\cdot e^{i\cdot(t_n\cdot x+\gamma2_n\cdot y)} + D_n\cdot e^{i\cdot(t_n\cdot x-\gamma2_n\cdot y)}\right]$$

$$u(x,y) := \left| \begin{array}{ll} u1(x,y) & \text{if } y > f(x) \\ u2(x,y) & \text{otherwise} \end{array} \right.$$

$Nx := 125$ $\quad Ny := 125$ $\quad nx := 0..Nx$ $\quad ny := 0..Ny$ $\quad b := \frac{T}{2}$

$xx_{nx} := 3T\cdot\frac{nx}{Nx}$ $\qquad yy_{ny} := -b + 7\cdot b\cdot\frac{ny}{Ny}$

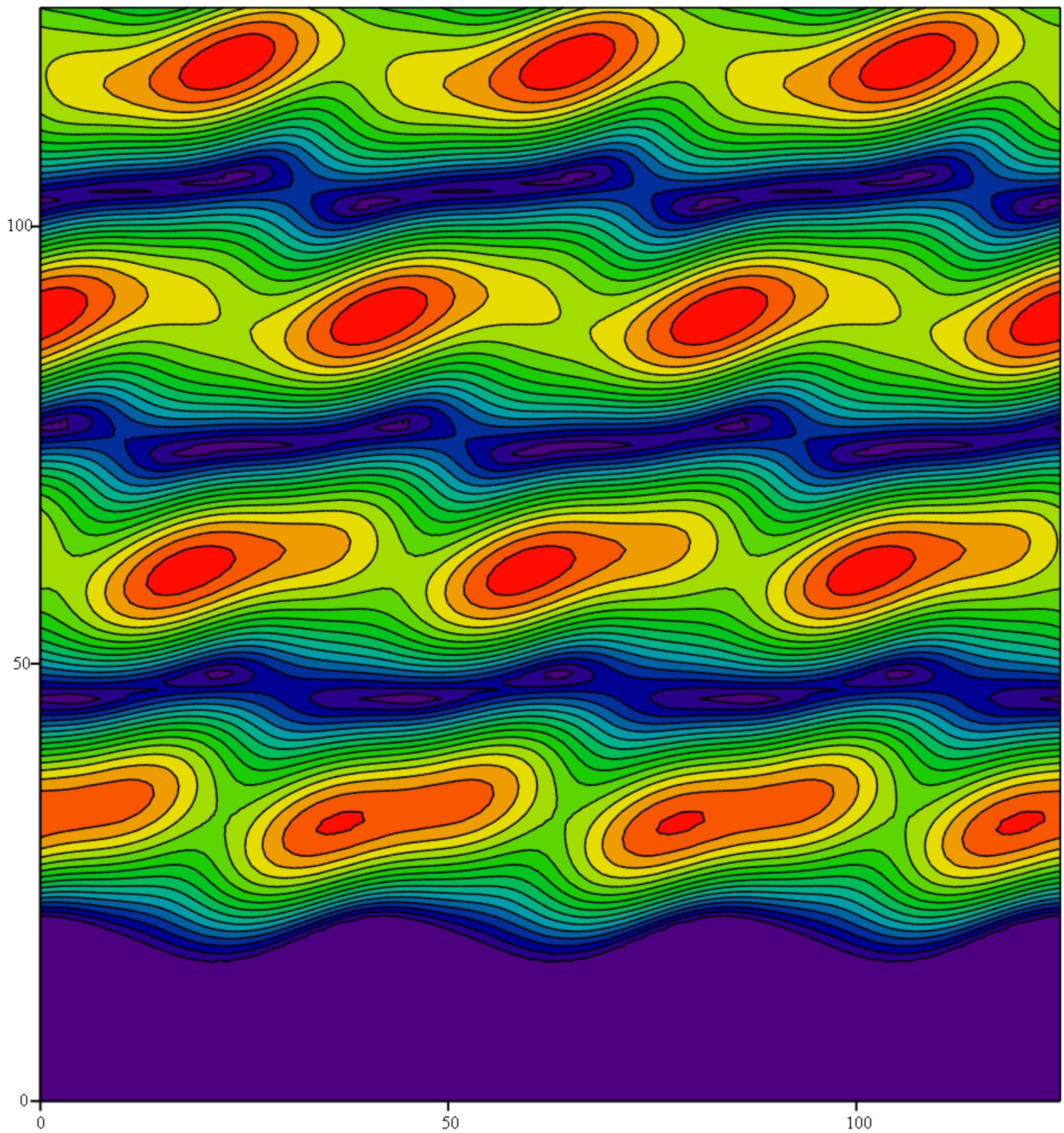
$xx^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.024	0.048	0.072	0.096	0.12	0.144	0.168	0.192	0.216

$yy^T =$

	0	1	2	3	4	5	6	7	8
0	-0.5	-0.472	-0.444	-0.416	-0.388	-0.36	-0.332	-0.304	-0.276

$U_{ny,nx} := u(xx_{nx}, yy_{ny})$ $\qquad V_{ny,nx} := \left|U_{ny,nx}\right|$



V^T