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Collocation method for the electromagnetic scattering problems

A.A.Bykov 2002

$$N := 17 \quad N2 := N \cdot 2 \quad n := 0 \dots N2 \quad j_n := n - N \quad M := N2 + 1$$

$$\lambda := 0.8 \quad T := 1 \quad k_0 := \frac{2 \cdot \pi}{\lambda}$$

$$\varepsilon_1 := 1 \quad \varepsilon_2 := 7.5 \quad k_1 := k_0 \cdot \sqrt{\varepsilon_1} \quad k_2 := k_0 \cdot \sqrt{\varepsilon_2} \quad \phi \equiv \frac{0}{180} \cdot \pi$$

$$\mu := k_1 \cdot T \cdot \sin(\phi) \quad t_n := \frac{\mu + 2 \cdot \pi \cdot j_n}{T} \quad \gamma_{1n} := \sqrt{k_1^2 - (t_n)^2} \quad \gamma_{2n} := \sqrt{k_2^2 - (t_n)^2}$$

$$\gamma_1^T =$$

	12	13	14	15	16	17
0	30.418i	23.874i	17.135i	9.81i	4.712	7.854

$$\gamma_2^T =$$

	0	1	2	3	4	5
0	104.626i	98.203i	91.761i	85.294i	78.799i	72.265i

$$\alpha := 0.08 \quad f(x) := \alpha \cdot \left(\cos\left(2 \cdot \pi \cdot \frac{x}{T}\right) + 0 \cdot \cos\left(4 \cdot \pi \cdot \frac{x}{T}\right) \right) \quad ff(x) := \frac{d}{dx} f(x)$$

$$K := 2 \cdot N \quad k := 0 \dots K \quad \xi_k := T \cdot \frac{k}{K + 1} \quad \psi_k := f(\xi_k) \quad i := \sqrt{-1}$$

$$nnnx_k := \frac{-ff(\xi_k)}{1 + ff(\xi_k)^2} \quad nnny_k := \frac{1}{1 + ff(\xi_k)^2}$$

$$P1_{k,n} := e^{i \cdot (t_n \cdot \xi_k + \gamma_{1n} \cdot \psi_k)}$$

$$P2_{k,n} := nnnx_k \cdot i \cdot t_n \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma_{1n} \cdot \psi_k)} + nnny_k \cdot i \cdot \gamma_{1n} \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma_{1n} \cdot \psi_k)}$$

$$Q1_{k,n} := -e^{i \cdot (t_n \cdot \xi_k - \gamma_{2n} \cdot \psi_k)}$$

$$Q2_{k,n} := -nnnx_k \cdot i \cdot t_n \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma_{2n} \cdot \psi_k)} + nnny_k \cdot i \cdot \gamma_{2n} \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma_{2n} \cdot \psi_k)}$$

$$B_n := 0 \quad B_N := 1 \quad B_{N-1} := 2 \quad B_{N+1} := 3 \quad C_n := 0$$

$$G1_k := -1 \cdot \left[\sum_n e^{i \cdot (t_n \cdot \xi_k - \gamma 1_n \cdot \psi_k)} \cdot B_n \right] + \sum_n e^{i \cdot (t_n \cdot \xi_k + \gamma 2_n \cdot \psi_k)} \cdot C_n$$

$$G2_k := \sum_n \left(-nnnx_k \cdot i \cdot t_n + nnny_k \cdot i \cdot \gamma 1_n \right) \cdot e^{i \cdot (t_n \cdot \xi_k - \gamma 1_n \cdot \psi_k)} \cdot B_n + \sum_n \left(nnnx_k \cdot i \cdot t_n + nnny_k \cdot i \cdot \gamma 2_n \right) \cdot e^{i \cdot (t_n \cdot \xi_k + \gamma 2_n \cdot \psi_k)} \cdot C_n$$

$$AA_{k,n} := P1_{k,n} \qquad xAA_{k,M+n} := Q1_{k,n} \qquad xAA_{M+k,n} := P2_{k,n} \qquad xAA_{M+k,M+n} := Q2_{k,n}$$

$$BB_k := G1_k \quad xBB_{M+k} := G2_k$$

$$G1^T =$$

	0	1	2	3
0	-5.458+2.428i	-5.465+2.196i	-5.346+1.843i	-5.091+1.392i

$$G2^T =$$

	0	1	2	3
0	13.29+28.261i	12.34+27.392i	11.54+25.366i	10.944+22.546i

$$BB^T =$$

	0	1	2	3
0	-5.458+2.428i	-5.465+2.196i	-5.346+1.843i	-5.091+1.392i

$$Sol := AA^{-1} \cdot BB$$

$$Sol^T =$$

	0	1	2
0	1.337·10 ⁻⁴ +6.155i·10 ⁻⁴	1.626·10 ⁻⁴ +6.899i·10 ⁻⁴	2.135·10 ⁻⁴ +8.732i·10 ⁻⁴

$$A_n := Sol_n \qquad D_n := 0 \cdot Sol_{0M+n}$$

$$A^T =$$

	0	1	2
0	1.337·10 ⁻⁴ +6.155i·10 ⁻⁴	1.626·10 ⁻⁴ +6.899i·10 ⁻⁴	2.135·10 ⁻⁴ +8.732i·10 ⁻⁴

$$D^T =$$

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0

$$ma_{n,0} := \left| A_n \right| \qquad md_{n,0} := \left| D_n \right| \qquad ma_{n,1} := \gamma 1_n \qquad md_{n,1} := \gamma 2_n$$

$\mathbf{ma}^T =$

	0	1	2	3	4	5
0	6.299·10 ⁻⁴	7.089·10 ⁻⁴	8.99·10 ⁻⁴	1.365·10 ⁻³	2.077·10 ⁻³	3.197·10 ⁻³
1	106.525i	100.224i	93.92i	87.613i	81.303i	74.988i

$\mathbf{md}^T =$

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	104.626i	98.203i	91.761i	85.294i	78.799i	72.265i

$$\sum_n \operatorname{Re}(\gamma_{1n}) \cdot \left(\left|A_n\right|\right)^2 + \sum_n \operatorname{Re}(\gamma_{2n}) \cdot \left(\left|D_n\right|\right)^2 = 69.115$$

$$\sum_n \operatorname{Re}(\gamma_{1n}) \cdot \left(\left|B_n\right|\right)^2 + \sum_n \operatorname{Re}(\gamma_{2n}) \cdot \left(\left|C_n\right|\right)^2 = 69.115$$

$$u1(x,y) := \sum_n \left[A_n \cdot e^{i \cdot \left(t_n \cdot x + \gamma_{1n} \cdot y\right)} + B_n \cdot e^{i \cdot \left(t_n \cdot x - \gamma_{1n} \cdot y\right)} \right]$$

$$u2(x,y) := \sum_n \left[C_n \cdot e^{i \cdot \left(t_n \cdot x + \gamma_{2n} \cdot y\right)} + D_n \cdot e^{i \cdot \left(t_n \cdot x - \gamma_{2n} \cdot y\right)} \right]$$

$$u(x,y) := \left\{ \begin{array}{l} u1(x,y) \text{ if } y > f(x) \\ u2(x,y) \text{ otherwise} \end{array} \right.$$

$$N_x := 125 \qquad N_y := 125 \qquad n_x := 0 .. N_x \qquad n_y := 0 .. N_y \qquad b := \frac{T}{2}$$

$$xx_{n_x} := 3T \cdot \frac{n_x}{N_x} \qquad yy_{n_y} := -b + 7 \cdot b \cdot \frac{n_y}{N_y}$$

$\mathbf{xx}^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.024	0.048	0.072	0.096	0.12	0.144	0.168	0.192	0.216

$\mathbf{yy}^T =$

	0	1	2	3	4	5	6	7	8
0	-0.5	-0.472	-0.444	-0.416	-0.388	-0.36	-0.332	-0.304	-0.276

$$U_{n_y,n_x} := u\Bigl(xx_{n_x},yy_{n_y}\Bigr) \qquad V_{n_y,n_x} := \left|U_{n_y,n_x}\right|$$

