

0.0.1. Многослойный планарный волновод

$$v'' + k_0^2 \varepsilon(x) v - \gamma^2 v = 0,$$

$$\varepsilon(x) = \begin{cases} \varepsilon_3, 0 \leq x < b_1, \\ \varepsilon_2, b_1 \leq x < b_2, \\ \varepsilon_1, b_2 \leq x < +\infty, \end{cases}$$

$$v(x) = \begin{cases} C_1 \cos(\sqrt{k_0^2 \varepsilon_3 - \gamma^2} x), & 0 \leq x < b_1, \\ C_2 \sinh(\sqrt{\gamma^2 - k_0^2 \varepsilon_2} (x - b_1)) + C_3 \cosh(\sqrt{\gamma^2 - k_0^2 \varepsilon_2} (x - b_1)), & b_1 \leq x < b_2, \\ C_4 e^{-\sqrt{\gamma^2 - k_0^2 \varepsilon_1} (x - b_2)}, & b_2 \leq x < +\infty, \end{cases}$$

$$\begin{cases} C_1 \cos(\sqrt{k_0^2 \varepsilon_3 - \gamma^2} b_1) = C_3, \\ -C_1 \sqrt{k_0^2 \varepsilon_3 - \gamma^2} \sin(\sqrt{k_0^2 \varepsilon_3 - \gamma^2} b_1) = C_2 \sqrt{\gamma^2 - k_0^2 \varepsilon_2}, \\ C_2 \sinh(\sqrt{\gamma^2 - k_0^2 \varepsilon_2} (b_2 - b_1)) + C_3 \cosh(\sqrt{\gamma^2 - k_0^2 \varepsilon_2} (b_2 - b_1)) = C_4, \\ C_2 \sqrt{\gamma^2 - k_0^2 \varepsilon_2} \cosh(\sqrt{\gamma^2 - k_0^2 \varepsilon_2} (b_2 - b_1)) + C_3 \sqrt{\gamma^2 - k_0^2 \varepsilon_2} \sinh(\sqrt{\gamma^2 - k_0^2 \varepsilon_2} (b_2 - b_1)) = -\sqrt{\gamma^2 - k_0^2 \varepsilon_1} C_4 \end{cases}$$

$$\left| \begin{array}{cccc} \cos(\sqrt{k_0^2 \varepsilon_3 - \gamma^2} b_1) & 0 & -1 & 0 \\ -\sqrt{k_0^2 \varepsilon_3 - \gamma^2} \sin(\sqrt{k_0^2 \varepsilon_3 - \gamma^2} b_1) & -\sqrt{\gamma^2 - k_0^2 \varepsilon_2} & 0 & 0 \\ 0 & \sinh(\sqrt{\gamma^2 - k_0^2 \varepsilon_2} (b_2 - b_1)) & \cosh(\sqrt{\gamma^2 - k_0^2 \varepsilon_2} (b_2 - b_1)) & -1 \\ 0 & \sqrt{\gamma^2 - k_0^2 \varepsilon_2} \cosh(\sqrt{\gamma^2 - k_0^2 \varepsilon_2} (b_2 - b_1)) & \sqrt{\gamma^2 - k_0^2 \varepsilon_2} \sinh(\sqrt{\gamma^2 - k_0^2 \varepsilon_2} (b_2 - b_1)) & \sqrt{\gamma^2 - k_0^2 \varepsilon_1} \end{array} \right|$$