Inverse fourier transform and the image reconstruction

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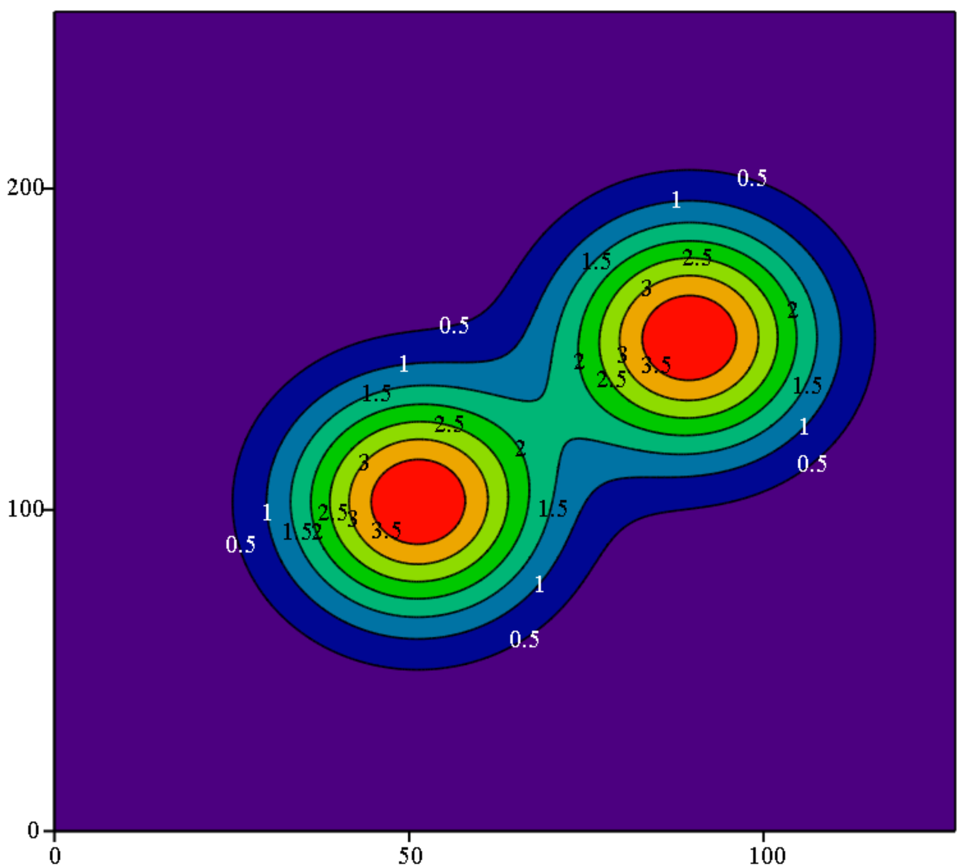
1. Image

$N := 2^8 \quad n := 0 .. N-1 \quad n1 := 0 .. N \qquad x_n := \frac{n}{N}$

$M := 2^7 \quad m := 0 .. M-1 \quad m1 := 0 .. N \qquad y_m := \frac{m}{M}$

$$fu(x, x0, y, y0, \sigma) := \frac{1}{\sqrt{2\cdot\pi\cdot\sigma}} \cdot e^{\frac{-(x-x0)^2-(y-y0)^2}{2\cdot\sigma^2}}$$

$fuu(x, y) := fu(x, 0.4, y, 0.4, 0.1) + fu(x, 0.6, y, 0.7, 0.1) \qquad u_{m,n} := fuu(x_n, y_m)$



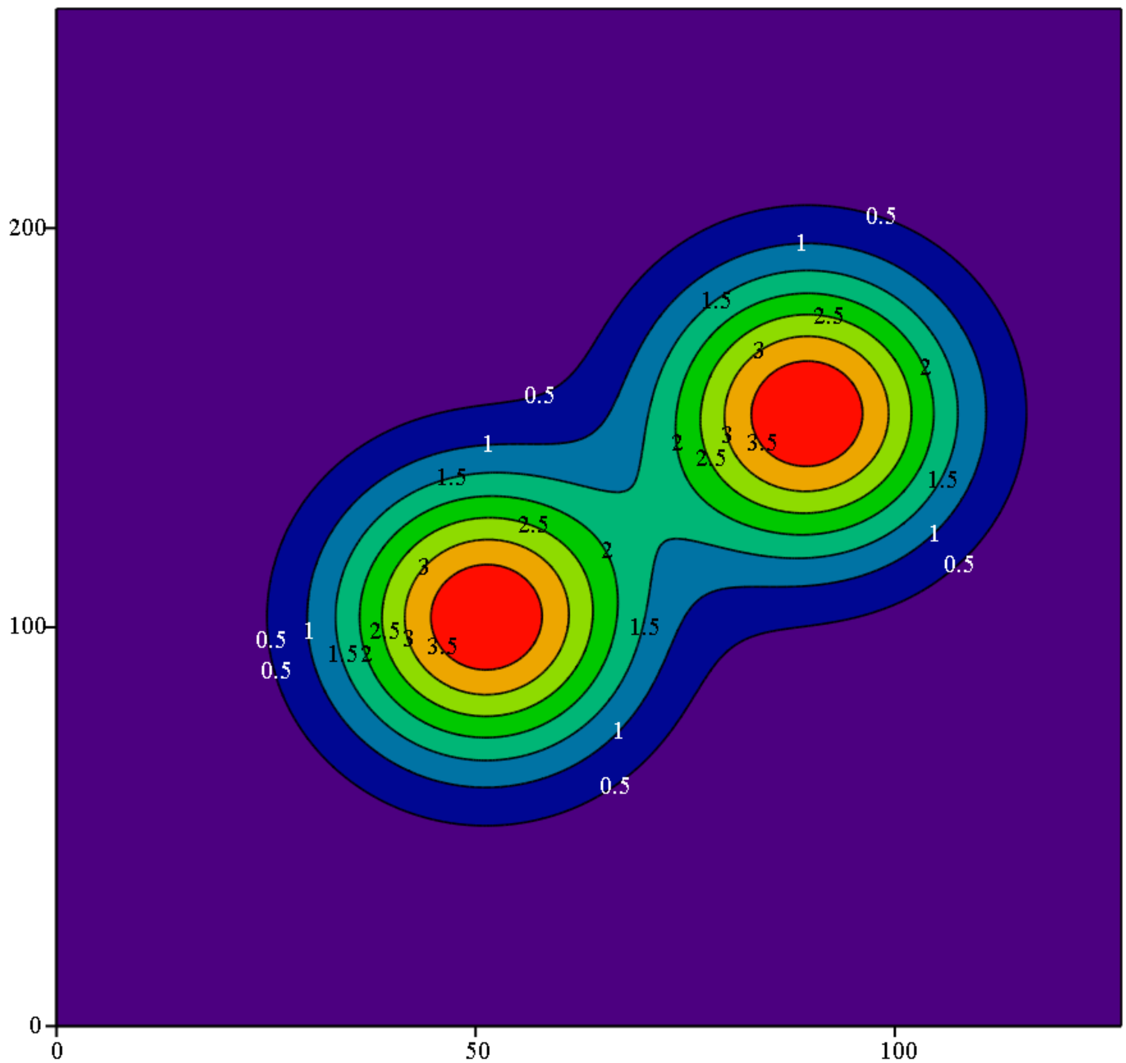
u

2. Fourier transform of the image

$U := \mathrm{cfft}(u)$

$U1 := \mathrm{CFFT}(U) \cdot \sqrt{M \cdot N}$

$\mathrm{cols}(U1) = 256$ $\mathrm{rows}(U1) = 128$



U1

3. Apparatus function

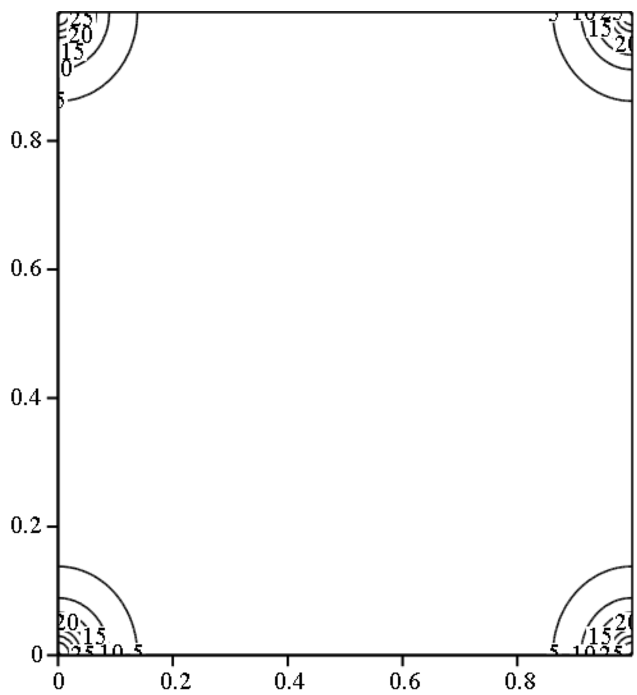
$\sigma 1 := 0.05$

$$fw1(x,y) := \frac{1}{(\pi \cdot \sigma 1)^2} \cdot \frac{1}{1 + \left(\frac{x}{\sigma 1}\right)^2 + \left(\frac{y}{\sigma 1}\right)^2}$$

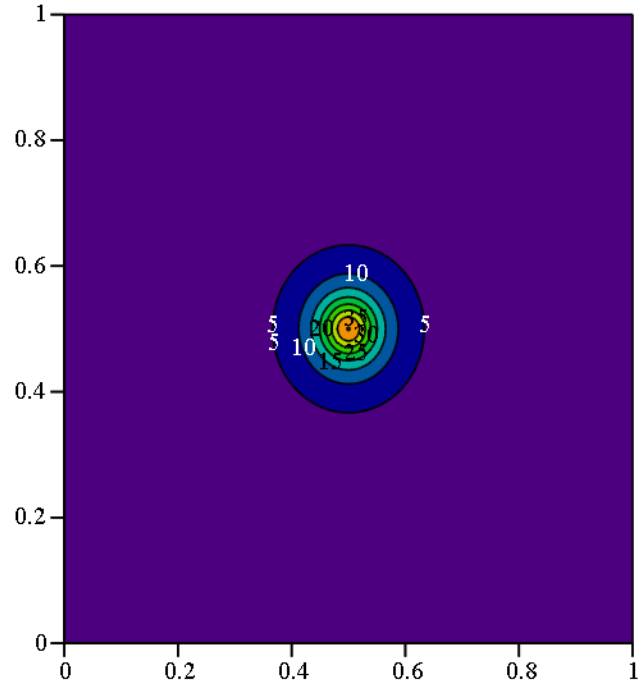
To make the apparatus function periodic

$fw2(x,y) := fw1(x,y) + fw1(1-x,y) + fw1(x,1-y) + fw1(1-x,1-y)$

$fw3(x,y) := fw1(x-0.5,y-0.5)$



fw2



fw3

$w_{m,n} := fw2\left(x_n, y_m\right)$

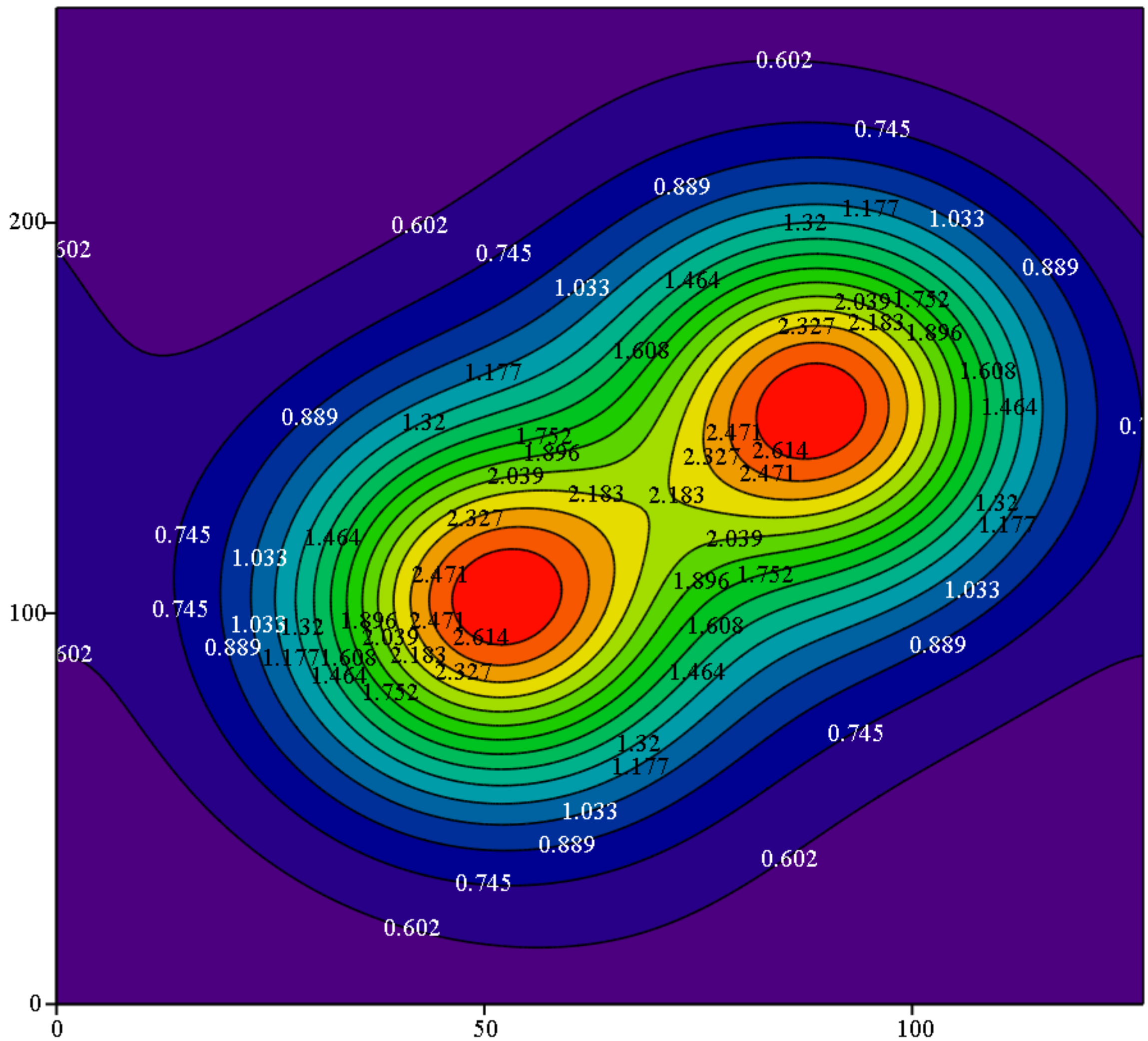
$W := cfft(w)$

Convolution

$V_{m,n} := U_{m,n} \cdot W_{m,n}$

$v2 := CFFT(V)$

$$v_{m,n} := \frac{v2_{m,n}}{\sqrt{M \cdot N}} \cdot \sqrt{M \cdot N}$$



v

Spam

$SPAM := rnorm(N \cdot M, 0, 1)$

$v1_{m,n} := v_{m,n} + 0.03 \cdot SPAM_{m+n \cdot M}$

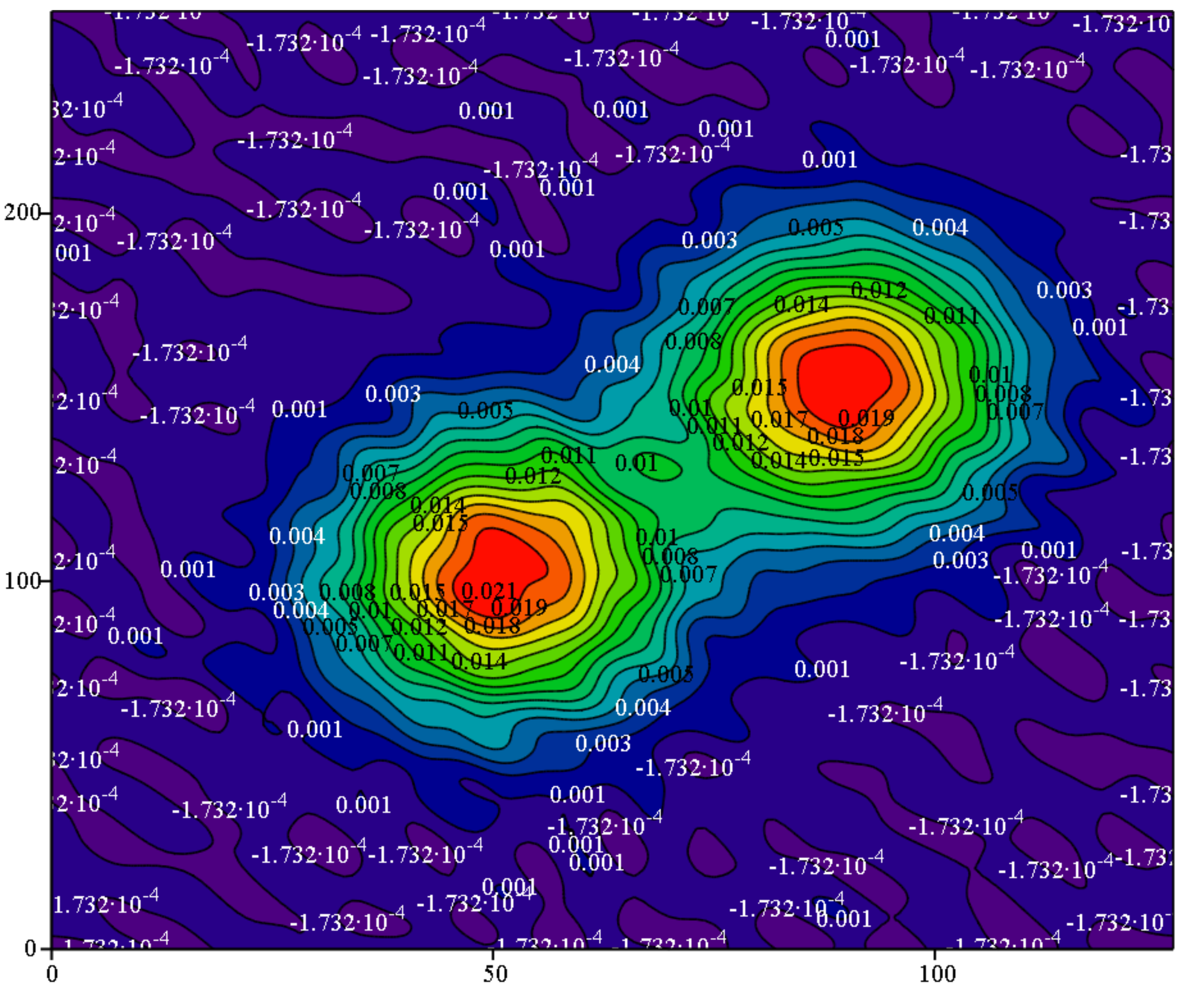
$V1 := cfft(v1)$

Regularization

$\alpha := 1.0 \cdot 10^{-3}$ $\beta := 1.0 \cdot 10^{-2}$

$$U1_{m,n} := \frac{V1_{m,n} \cdot \overline{W_{m,n}}}{\overline{W_{m,n}} W_{m,n} + \alpha + \beta \cdot \left(m^2 + n^2\right)}$$

$u1p := CFFT(U1)$ $u1k_{m,n} := u1p_{m,n} \cdot \sqrt{M \cdot N}$

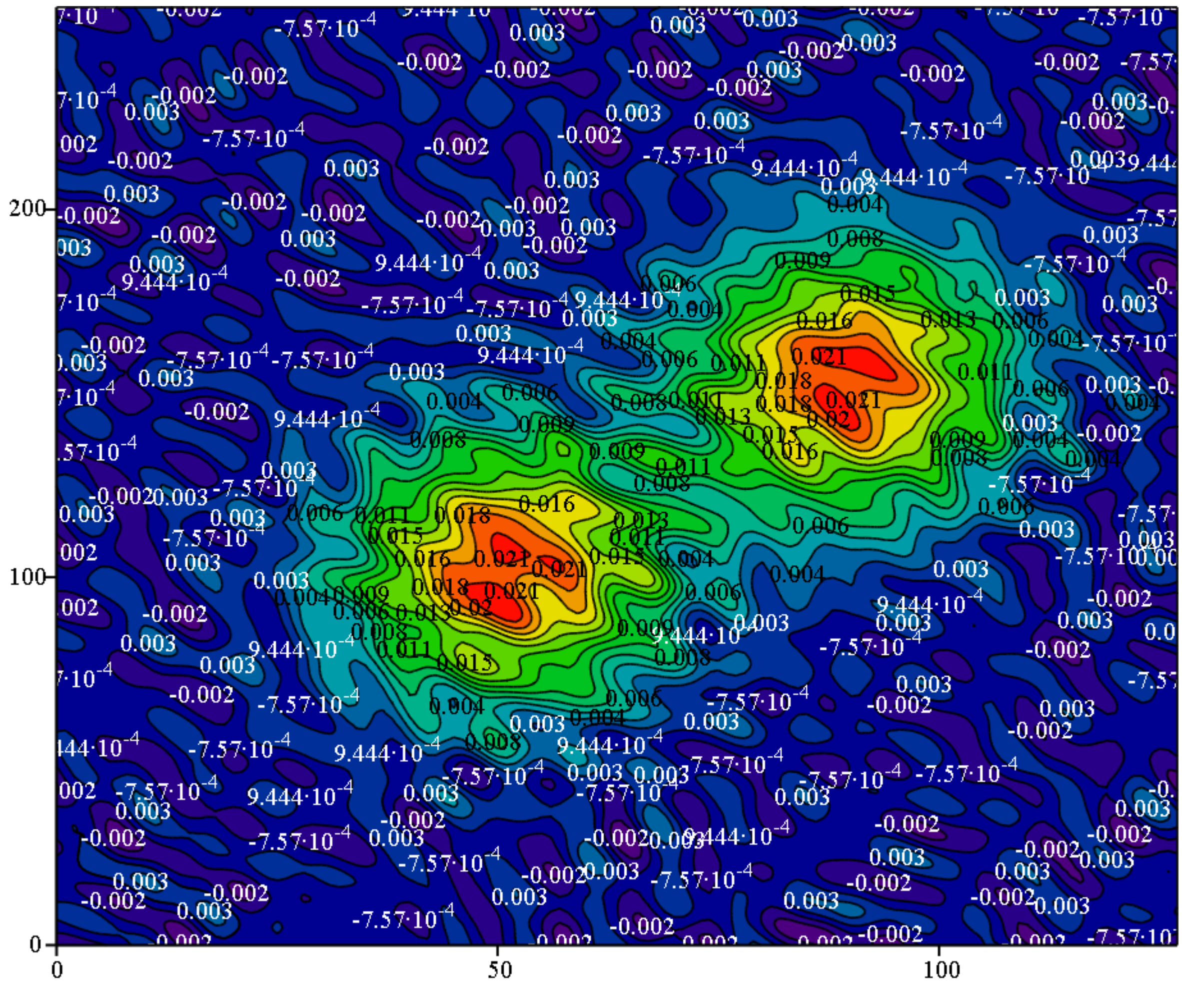


u1k

Regularization

$\alpha := 1.0 \cdot 10^{-3}$ $\quad\beta := 1.0 \cdot 10^{-3}$ $\quad U1_{m,n} := \dfrac{V1_{m,n} \cdot \overline{W_{m,n}}}{\overline{W_{m,n}} W_{m,n} + \alpha + \beta \cdot \left(m^2 + n^2\right)}$

$u1p := \mathrm{CFFT}(U1) \quad u1k_{m,n} := u1p_{m,n} \cdot \sqrt{M \cdot N}$



u1k