Galerkin-method for electromagnetic scattering problem in waveguides

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$$\int_{y1}^{y2} \sin\left(\frac{\pi\cdot n}{b}\cdot x\right)\cdot\sin\left(\frac{\pi\cdot m}{b}\cdot x\right)dx \to -b\cdot\frac{-n\cdot\cos\left(\frac{1}{b}\cdot y2\cdot\pi\cdot n\right)\cdot\sin\left(\frac{1}{b}\cdot y2\cdot\pi\cdot m\right)+m\cdot\sin\left(\frac{1}{b}\cdot y2\cdot\pi\cdot n\right)\cdot\cos\left(\frac{1}{b}\cdot y2\cdot\pi\cdot m\right)}{\pi\cdot(-n+m)\cdot(n+m)}+b\cdot\frac{-n\cdot\cos\left(\frac{1}{b}\cdot y1\cdot\pi\cdot n\right)\cdot\sin\left(\frac{1}{b}\cdot y1\cdot\pi\cdot m\right)+m\cdot\sin\left(\frac{1}{b}\cdot y1\cdot\pi\cdot n\right)\cdot\cos\left(\frac{1}{b}\cdot y1\cdot\pi\cdot\right.}{\pi\cdot(-n+m)\cdot(n+m)}$$

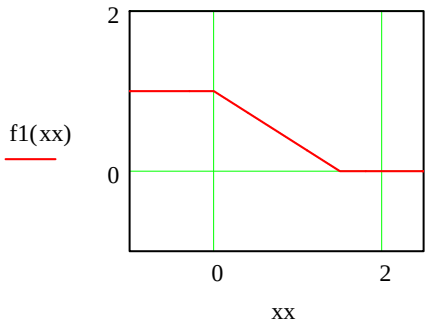
$$\int_{y1}^{y2} \sin\left(\frac{\pi\cdot n}{b}\cdot x\right)\cdot\sin\left(\frac{\pi\cdot n}{b}\cdot x\right)dx \to \frac{-1}{2}\cdot\frac{\cos\left(\frac{1}{b}\cdot y2\cdot\pi\cdot n\right)\cdot\sin\left(\frac{1}{b}\cdot y2\cdot\pi\cdot n\right)\cdot b-y2\cdot\pi\cdot n}{\pi\cdot n}+\frac{1}{2}\cdot\frac{\cos\left(\frac{1}{b}\cdot y1\cdot\pi\cdot n\right)\cdot\sin\left(\frac{1}{b}\cdot y1\cdot\pi\cdot n\right)\cdot b-y1\cdot\pi\cdot n}{\pi\cdot n}$$

$$\mathrm{bra}(m,n,y1,y2,b):=\begin{cases} b\cdot\dfrac{\left(-n\cdot\cos\left(\frac{1}{b}\cdot y2\cdot\pi\cdot n\right)\cdot\sin\left(\frac{1}{b}\cdot y2\cdot\pi\cdot m\right)+m\cdot\sin\left(\frac{1}{b}\cdot y2\cdot\pi\cdot n\right)\cdot\cos\left(\frac{1}{b}\cdot y2\cdot\pi\cdot m\right)\right)}{\pi\cdot(n-m)\cdot(n+m)}+b\cdot\dfrac{\left(n\cdot\cos\left(\frac{1}{b}\cdot y1\cdot\pi\cdot n\right)\cdot\sin\left(\frac{1}{b}\cdot y1\cdot\pi\cdot m\right)-m\cdot\sin\left(\frac{1}{b}\cdot y1\cdot\pi\cdot n\right)\cdot\cos\left(\frac{1}{b}\cdot y1\cdot\pi\cdot m\right)\right)}{\pi\cdot(n-m)\cdot(n+m)} & \text{if } n \\ \dfrac{-1}{2}\cdot\dfrac{\left(\cos\left(\frac{1}{b}\cdot y2\cdot\pi\cdot n\right)\cdot\sin\left(\frac{1}{b}\cdot y2\cdot\pi\cdot n\right)\cdot b-y2\cdot\pi\cdot n\right)}{\pi\cdot n}+\dfrac{1}{2}\cdot\dfrac{\left(\cos\left(\frac{1}{b}\cdot y1\cdot\pi\cdot n\right)\cdot\sin\left(\frac{1}{b}\cdot y1\cdot\pi\cdot n\right)\cdot b-y1\cdot\pi\cdot n\right)}{\pi\cdot n} & \text{otherwise} \end{cases}$$

$i := \sqrt{-1}$ $\varepsilon 1 := 17 + i\cdot 0$ $\varepsilon 2 := 1$ $a := 1.5$ $b := 1$ $k0 := 6.5$

$nst := 2$ $N \equiv 10$

$$f1(x) := \begin{cases} b & \text{if } x < 0 \\ (a - x)\cdot\frac{b}{a} & \text{if } 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$$



$$ee(m,n,x) := \left(\varepsilon1\cdot bra(m+1,n+1,0,f1(x),b) + \varepsilon2\cdot bra(m+1,n+1,f1(x),b,b)\right)\cdot\frac{2}{b}$$

$N = 10$ $n := 0 .. N - 1$ $m := 0 .. N - 1$

$NN2 := N\cdot N\cdot 2$ $nn2 := 0 .. NN2 - 1$ $NN2 = 200$ $N2 := N\cdot 2$

$N4 := N\cdot 4$ $N3 := N\cdot 3$ $NN4 := N\cdot N\cdot 4$ $nn4 := 0 .. NN4 - 1$

$$\Lambda_n := \frac{\pi\cdot(n+1)}{b} \qquad g1_n := \sqrt{k0^2\cdot\varepsilon1 - (\Lambda_n)^2} \qquad g2_n := \sqrt{k0^2\cdot\varepsilon2 - (\Lambda_n)^2}$$

$rg1_n := Re(g1_n)$ $ig1_n := Im(g1_n)$ $rg2_n := Re(g2_n)$ $ig2_n := Im(g2_n)$

$rg1^T =$

	0	1	2	3	4	5	6	7	8
0	26.615	26.053	25.088	23.671	21.714	19.051	15.318	9.306	0

$rg2^T =$

	0	1	2	3	4	5	6	7	8	9
0	5.69	1.665	0	0	0	0	0	0	0	0

$g1^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	26.615	26.053	25.088	23.671	21.714	19.051	15.318	9.306	9.01i	16.392i	

$g2^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	5.69	1.665	6.825i	10.755i	14.3i	17.693i	21.009i	24.278i	27.517i	30.736i	

$K := 35 \qquad k := 0 .. K \qquad hx := \frac{a}{K} \qquad x_k := k \cdot hx \qquad hx = 0.043$

$L := 20 \qquad l := 0 .. L \qquad hy := \frac{b}{L} \qquad y_l := l \cdot hy \qquad hy = 0.05$

$y^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45

$x^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0.043	0.086	0.129	0.171	0.214	0.257	0.3	0.343	0.386

$Z7_{m \cdot 2+n \cdot N4, k} := 0 \qquad Z7_{m \cdot 2+1+n \cdot N4, k} := 0 \qquad Z7_{m \cdot 2+N2+n \cdot N4, k} := 0 \qquad Z7_{m \cdot 2+1+N2+n \cdot N4, k} := 0$

$D(t, Y) :=$

for $r \in 0 .. NN4 - 1$

$Y1_r \leftarrow 0$

for $p \in 0 .. N - 1$

for $q \in 0 .. N - 1$

$Y1_{p \cdot 2+q \cdot N4} \leftarrow Y_{p \cdot 2+q \cdot N4+N2}$

$Y1_{p \cdot 2+1+q \cdot N4} \leftarrow Y_{p \cdot 2+1+q \cdot N4+N2}$

for $p \in 0 .. N - 1$

for $q \in 0 .. N - 1$

$rQ_{p,q} \leftarrow 0$

$zQ_{p,q} \leftarrow 0$

for $p \in 0 .. N - 1$

$rQ_{p,p} \leftarrow rQ_{p,p} + (\Lambda_p)^2$

for $p \in 0 .. N - 1$

for $q \in 0 .. N - 1$

$rQ_{p,q} \leftarrow rQ_{p,q} - k0^2 \cdot Re(ee(p, q, t))$

$zQ_{\_,\_} \leftarrow zQ_{\_,\_} - k0^2 \cdot Im(ee(p, q, t))$

$p, q \qquad p, q$

for $p \in 0 .. N - 1$

  for $q \in 0 .. N - 1$

$$Y1_{p\cdot 2+1+q\cdot N4+N2} \leftarrow \sum_{r=0}^{N-1} \left(rQ_{p,r} \cdot Y_{r\cdot 2+1+q\cdot N4} + zQ_{p,r} \cdot Y_{r\cdot 2+q\cdot N4}\right)$$

$$Y1_{p\cdot 2+q\cdot N4+N2} \leftarrow \sum_{r=0}^{N-1} \left(rQ_{p,r} \cdot Y_{r\cdot 2+q\cdot N4} - zQ_{p,r} \cdot Y_{r\cdot 2+1+q\cdot N4}\right)$$

return Y1

$Y0_{m\cdot 2+n\cdot N4} := 0$ $Y0_{m\cdot 2+1+n\cdot N4} := 0$ $Y0_{m\cdot 2+N2+n\cdot N4} := 0$ $Y0_{m\cdot 2+1+N2+n\cdot N4} := 0$

$Y0_{n\cdot 2+n\cdot N4} := 1$ $Y0_{n\cdot 2+N2+n\cdot N4} := ig1_n$ $Y0_{n\cdot 2+1+N2+n\cdot N4} := -rg1_n$

$D7 := D(0, Y0)$

$Y0^T =$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

$D7^T =$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	1
0	0	-26.615	0	0	0	0	0	0	0	0	0	0	0	0	

```
slv(jj) := | for r ∈ 0 .. NN4 − 1
           |   Z_{r,0} ← Y0_r
           | for k1 ∈ 1 .. K
           |   x1 ← hx·(k1 − 1)
           |   x2 ← hx·k1
           |   for r ∈ 0 .. NN4 − 1
           |     Yic_r ← Z_{r,k1−1}
           |   Y2 ← rkfixed(Yic, x1, x2, jj, D)
           |   for r ∈ 0 .. NN4 − 1
           |     Z_{r,k1} ← Y2_{jj,r+1}
           | return Z
```

nst = 2

f := slv(nst)

f =

	0	1	2	3	4	5
0	1	0.418	-0.65	-0.962	-0.159	0.818
1	0	-0.908	-0.759	0.273	0.986	0.559
2	0	$-1.313\cdot10^{-5}$	$-3.823\cdot10^{-6}$	$2.434\cdot10^{-3}$	0.013	0.029
3	0	$1.248\cdot10^{-5}$	$5.095\cdot10^{-4}$	$2.372\cdot10^{-3}$	$6.839\cdot10^{-4}$	-0.017
4	0	$1.97\cdot10^{-5}$	$9.296\cdot10^{-6}$	$-3.587\cdot10^{-3}$	-0.019	-0.044
5	0	$-1.87\cdot10^{-5}$	$-7.639\cdot10^{-4}$	$-3.587\cdot10^{-3}$	$-1.445\cdot10^{-3}$	0.023
6	0	$-2.628\cdot10^{-5}$	$-1.907\cdot10^{-5}$	$4.662\cdot10^{-3}$	0.025	0.059
7	0	$2.49\cdot10^{-5}$	$1.018\cdot10^{-3}$	$4.839\cdot10^{-3}$	$2.72\cdot10^{-3}$	-0.027
8	0	$3.286\cdot10^{-5}$	$3.465\cdot10^{-5}$	$-5.634\cdot10^{-3}$	-0.03	-0.074
9	0	$-3.107\cdot10^{-5}$	$-1.271\cdot10^{-3}$	$-6.144\cdot10^{-3}$	$-4.701\cdot10^{-3}$	0.027
10	0	$-3.946\cdot10^{-5}$	$-5.756\cdot10^{-5}$	$6.479\cdot10^{-3}$	0.035	0.089
11	0	$3.721\cdot10^{-5}$	$1.525\cdot10^{-3}$	$7.522\cdot10^{-3}$	$7.602\cdot10^{-3}$	-0.023
12	0	$4.607\cdot10^{-5}$	$8.942\cdot10^{-5}$	$-7.169\cdot10^{-3}$	-0.04	-0.104
13	0	$-4.331\cdot10^{-5}$	$-1.778\cdot10^{-3}$	$-8.995\cdot10^{-3}$	-0.012	0.014
14	0	$-5.27\cdot10^{-5}$	$-1.319\cdot10^{-4}$	$7.679\cdot10^{-3}$	0.043	0.12
15	0	$4.936\cdot10^{-5}$	$2.031\cdot10^{-3}$	0.011	0.017	$1.991\cdot10^{-3}$

$Y_{nn4} := f_{nn4,K}$ $\quad ind := 0$

$Y^T =$

	0	1	2	3	4	5	6	7	8	9	10
0	$3\cdot10^{11}$	$7\cdot10^{10}$	$9\cdot10^{12}$	$5\cdot10^{11}$	$3\cdot10^{12}$	$7\cdot10^{11}$	$7\cdot10^{12}$	$5\cdot10^{11}$	$3\cdot10^{12}$	$6\cdot10^{11}$	$5\cdot10^{12}$

$A_{m,n} := Y_{m\cdot2+N2+n\cdot N4} + ig2_m\cdot Y_{m\cdot2+n\cdot N4} + rg2_m\cdot Y_{m\cdot2+1+n\cdot N4} + i\cdot\left(Y_{m\cdot2+1+N2+n\cdot N4} - rg2_m\cdot Y_{m\cdot2+n\cdot N4} + ig2_m\cdot Y_{m\cdot2+1+n\cdot N4}\right)$

$A1 := A^{-1}$ $\quad B_n := 0$ $\quad B_{ind} := -2\cdot i\cdot\left(rg2_{ind} + i\cdot ig2_{ind}\right)\cdot 1$ $\quad |A| = 1.505\times10^{73} - 3.311i\times10^{72}$

$C_n := \sum_m A1_{n,m}\cdot B_m$

$C^T =$

	0	1	2
0	0.072-0.092i	-5.825·10 -3-0.043i	-0.076-0.012i

$YY_{m,k} := \sum_n \left(f_{2\cdot m+n\cdot N4,k} + i\cdot f_{2\cdot m+1+n\cdot N4,k}\right)\cdot C_n$

$YYd_{m,k} := \sum_n \left(f_{2\cdot m+N2+n\cdot N4,k} + i\cdot f_{2\cdot m+1+N2+n\cdot N4,k}\right)\cdot C_n$

$R1_n := \left(2\cdot i\cdot g1_n\right)^{-1}\cdot\left(YYd_{n,0} + i\cdot g1_n\cdot YY_{n,0}\right)$

$R1^T =$

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0

$T1_n := \left(2\cdot i\cdot g1_n\right)^{-1}\cdot\left(-YYd_{n,0} + i\cdot g1_n\cdot YY_{n,0}\right)$

$T1^T =$

	0	1	2
0	0.072-0.092i	-5.825·10 -3-0.043i	-0.076-0.012i

$$R2_n := \left(2 \cdot i \cdot g2_n\right)^{-1} \cdot \left(YYd_{n,K} + i \cdot g2_n \cdot YY_{n,K}\right)$$

$R2^T =$

	0	1	2
0	0.194-0.385i	0.361-0.03i	-0.04+0.048i

$$T2_n := \left(2 \cdot i \cdot g2_n\right)^{-1} \cdot \left(-YYd_{n,K} + i \cdot g2_n \cdot YY_{n,K}\right)$$

$T2^T =$

	0	1	2
0	$1+8.135i \cdot 10^{-4}$	$7.143 \cdot 10^{-4}+3.588i \cdot 10^{-4}$	$5.017 \cdot 10^{-4}-5.43i \cdot 10^{-4}$

$$Wi := \sum_n \left[\left(\left|T2_n\right|\right)^2 \cdot rg2_n + \left(\left|R1_n\right|\right)^2 \cdot rg1_n\right] \qquad Wi = 5.692$$

$$Wr := \sum_n \left[\left(\left|R2_n\right|\right)^2 \cdot rg2_n + \left(\left|T1_n\right|\right)^2 \cdot rg1_n\right] \qquad Wr = 5.695$$

$YY =$

	0	1	2	3	4	5
0	0.072-0.092i	-0.053-0.104i	$-0.117+4.878i \cdot 10^{-3}$	-0.045+0.107i	0.077+0.086i	0.109-0.032
1	$-5.825 \cdot 10^{-3}-0.043i$	-0.041-0.014i	-0.03+0.032i	0.017+0.043i	$0.05+3.332i \cdot 10^{-3}$	0.026-0.047
2	-0.076-0.012i	-0.047+0.061i	0.031+0.069i	$0.072+2.763i \cdot 10^{-3}$	0.031-0.062i	-0.042-0.052
3	0.06+0.07i	0.091-0.014i	0.037-0.083i	-0.047-0.071i	$-0.078+2.938i \cdot 10^{-3}$	-0.035+0.062
4	-0.064-0.084i	$-0.106+8.895i \cdot 10^{-4}$	-0.063+0.083i	0.024+0.096i	0.083+0.036i	0.073-0.039
5	0.079+0.27i	0.251+0.127i	0.266-0.093i	0.12-0.252i	-0.091-0.257i	-0.242-0.113
6	0.319-0.114i	0.184-0.285i	-0.029-0.341i	-0.238-0.259i	-0.359-0.064i	-0.335+0.17
7	-0.079-0.097i	-0.11-0.059i	$-0.123-8.047i \cdot 10^{-3}$	-0.108+0.049i	-0.064+0.093i	$-5.662 \cdot 10^{-3}+0.111$
8	$0.013-7.095i \cdot 10^{-4}$	$0.019-1.236i \cdot 10^{-3}$	$0.027-5.475i \cdot 10^{-3}$	0.03-0.016i	0.025-0.024i	0.018-0.028
9	$-3.384 \cdot 10^{-3}-1.658i \cdot 10^{-3}$	$-6.873 \cdot 10^{-3}-3.135i \cdot 10^{-3}$	$-0.013-2.301i \cdot 10^{-3}$	$-0.015+3.31i \cdot 10^{-3}$	$-0.013+7.048i \cdot 10^{-3}$	$-0.012+8.75i \cdot 10^{-}$
10						
11						

$$\Phi_{l,n} := \sqrt{\frac{2}{b}} \cdot \sin\left[\frac{\pi \cdot (n+1)}{b} \cdot y_l\right] \qquad u_{l,k} := \sum_n YY_{n,k} \cdot \Phi_{l,n} \qquad v_{l,k} := \left|u_{l,k}\right|$$

$$w_{l+k\cdot L,0} := x_k \qquad w_{l+k\cdot L,1} := y_l \qquad w_{l+k\cdot L,2} := v_{l,k}$$

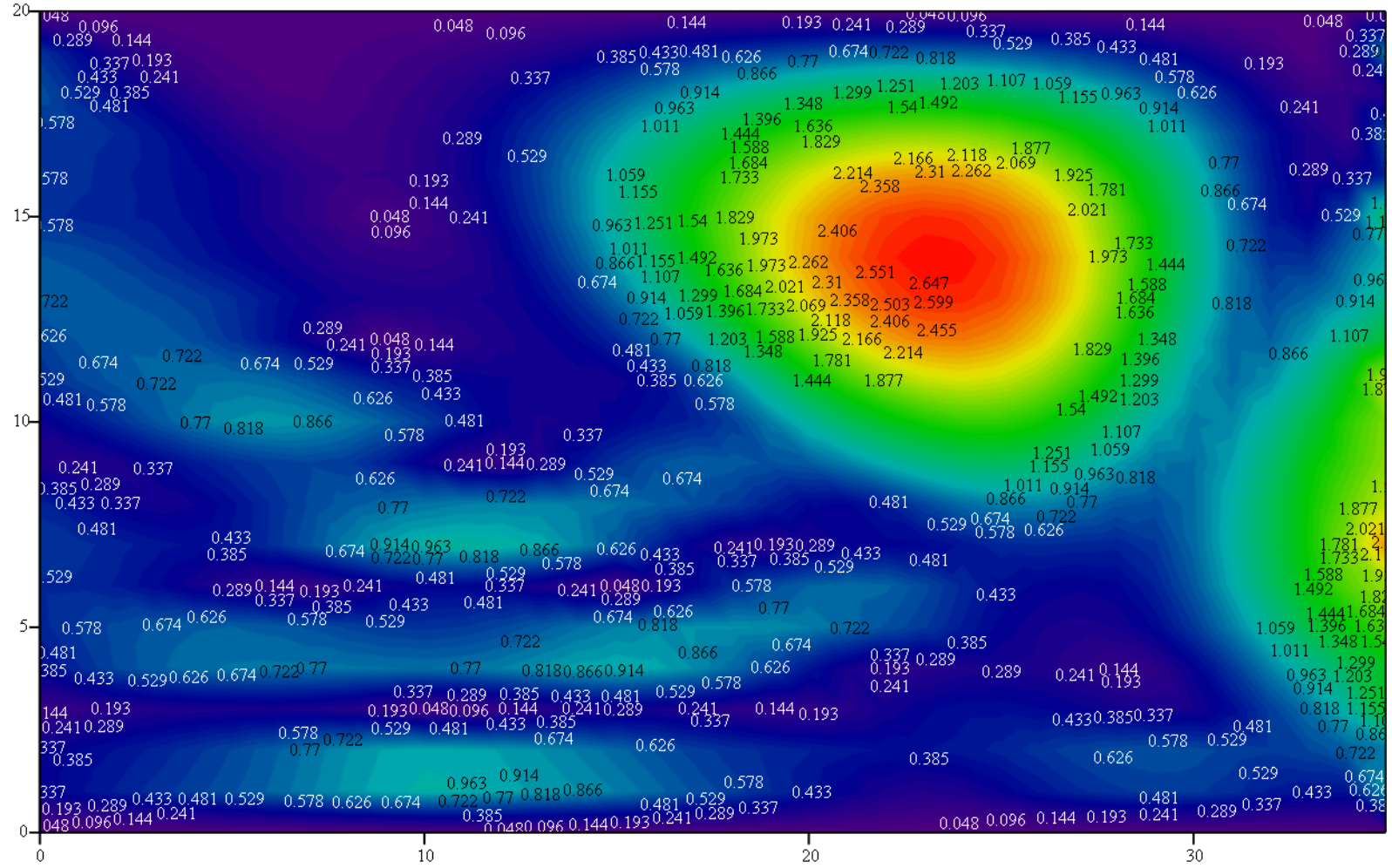
WRITEPRN("u.dat") := w

u =

	0	1	2
0	0	0	0
1	0.351-0.041i	0.298-0.299i	0.07-0.484i
2	0.342+0.038i	0.327-0.208i	0.153-0.411i
3	-0.032+0.112i	0.016+0.159i	0.129+0.159i
4	-0.401-0.033i	-0.41+0.323i	-0.103+0.621i
5	-0.371-0.352i	-0.582+0.036i	-0.402+0.499i
6	0.065-0.513i	-0.317-0.411i	-0.47-0.059i
7	0.496-0.291i	0.151-0.528i	-0.202-0.461i
8	0.494+0.107i	0.366-0.198i	0.141-0.307i
9	0.069+0.235i	0.097+0.188i	0.168+0.205i
10	-0.315-0.073i	-0.392+0.17i	-0.218+0.502i
11	-0.21-0.485i	-0.568-0.264i	-0.648+0.238i
12	0.32-0.521i	-0.179-0.652i	-0.629-0.345i
13	0.722-0.084i	0.434-0.556i	-0.096-0.668i
14	0.524+0.43i	0.659-0.027i	0.446-0.416i
15	-0.12+0.539i	0.265+0.444i	0.47+0.151i

v =

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	
1	0.353	0.422	0.489	0.55	0.607	0.664	0.73	0.807	0.886	0.948	0.976	0.967	0.
2	0.344	0.387	0.438	0.496	0.56	0.629	0.704	0.785	0.863	0.923	0.95	0.942	(
3	0.117	0.16	0.205	0.235	0.251	0.261	0.266	0.254	0.21	0.135	0.044	0.053	0.
4	0.402	0.522	0.63	0.709	0.756	0.776	0.783	0.785	0.781	0.772	0.767	0.78	0.
5	0.511	0.583	0.641	0.686	0.715	0.722	0.703	0.664	0.616	0.577	0.56	0.577	0.
6	0.517	0.519	0.474	0.395	0.305	0.216	0.132	0.116	0.211	0.313	0.369	0.372	0.
7	0.575	0.549	0.503	0.453	0.425	0.451	0.537	0.664	0.805	0.928	1.008	1.034	1.
8	0.506	0.416	0.338	0.301	0.329	0.417	0.53	0.63	0.698	0.742	0.78	0.813	0.
9	0.245	0.211	0.265	0.363	0.459	0.536	0.592	0.617	0.586	0.484	0.33	0.172	0.
10	0.323	0.427	0.547	0.673	0.79	0.872	0.9	0.874	0.807	0.714	0.599	0.47	0.
11	0.529	0.626	0.69	0.73	0.758	0.771	0.747	0.673	0.567	0.473	0.415	0.375	0.
12	0.612	0.677	0.717	0.715	0.66	0.563	0.442	0.309	0.168	0.036	0.105	0.201	0.
13	0.727	0.706	0.675	0.643	0.601	0.539	0.455	0.369	0.305	0.275	0.27	0.277	0.
14	0.677	0.659	0.61	0.538	0.46	0.391	0.33	0.268	0.208	0.169	0.163	0.207	0.
15	0.552	0.517	0.494	0.466	0.419	0.349	0.267	0.181	0.096	0.047	0.119	0.238	0.
16	0.585	0.547	0.497	0.45	0.406	0.346	0.264	0.186	0.142	0.149	0.205	0.304	0.
17	0.544	0.557	0.527	0.448	0.34	0.237	0.153	0.103	0.103	0.141	0.205	0.299	0.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